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AN INTRODUCTION TO
NUCLEAR WEAPONS(u)

By Samuel Glasstone and
Leslie M. Redman

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FOREWORD

While this revision of WASH-1037 and the original issuance have been published as Headquarters, U. S. Atomic Energy Commission documents, they have as their genesis two 1954 Los Alamos Scientific Laboratory reports, LA 1632 and LA 1633, both entitled "Weapons Activities of LASL." We are indebted to LASL for these early reports. Although the previous LA and WASH reports received extensive use as basic handbooks on the principles of nuclear weapons development and technology, they were not to be considered as technical guides for designing weapons. Similarly, this revision of WASH-1037 is not to serve as such a guide.

In the preparation of this document, Dr. Samuel Glasstone has reviewed, coordinated, and edited data provided by members of the Los Alamos Scientific Laboratory, the Lawrence Livermore Laboratory, the Sandia Laboratories, and the Defense Nuclear Agency. The exceptional cooperation of these organizations and the outstanding work of Dr. Glasstone have permitted the Atomic Energy Commission to publish this revision to WASH-1037. Further, we wish to recognize the very significant contributions of Dr. Leslie M. Redman in the preparation of this issuance.

This publication contains highly sensitive nuclear weapon design information of significance to our national defense and security. Viewers are enjoined to ensure its proper security protection at all times.

Edward B. Giller
Major General, USAF
Assistant General Manager
  for Military Application
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PREFACE

The overall design of a nuclear weapon is a complex operation involving a cooperative effort among mathematicians, scientists, and engineers in several areas of specialization. In spite of the complexity of these weapons, their design and operation are based on certain fundamental physical principles. One of the purposes of this report is to provide a description of these principles in fairly elementary terms. The mathematical aspects of weapon design have been omitted or oversimplified, since the emphasis here is placed on the physical phenomena rather than on the mathematical techniques used in treating them.

Another objective is to describe how the ideas used in weapons design have changed over the years. The earliest atomic bombs weighed some 8000 pounds and gave an explosive energy yield in the vicinity of 20 kilotons TNT equivalent.

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A semihistorical approach is used to show how such improvements in efficiency have been attained and indications are given of new directions in which further developments appear possible. It will be apparent that advances in weapons design have been continuous in the past and may be expected to be so in the future.

In preparing this report, we have received invaluable assistance from members of the Los Alamos Scientific Laboratory, Lawrence Livermore Laboratory, Sandia Laboratories (Albuquerque and Livermore), the Defense Nuclear Agency (Headquarters and Field Command), and the AEC Division of Military Application. To all the people who helped us by supplying information and/or providing constructive reviews of the draft material, we wish to express our great indebtedness.

Samuel Glasstone
Leslie M. Redman

Chapter 1

PRINCIPLES OF NUCLEAR ENERGY RELEASE

INTRODUCTION

Mass-Energy Equivalence

1.01 Any nuclear reaction in which there is a net decrease of mass, i.e., in which the total mass of the products is less than that of the interacting nuclei or nuclear particles, will be accompanied by a liberation of energy. The amount of energy released E (in ergs) is related to the net decrease of mass m (in grams) by the Einstein equation,

$$E = mc^2, \qquad (1.1)$$

where c is the velocity of light, i.e., 3.00×10^{10} cm/sec. In the study of nuclear reactions it is the common practice to state energies in electron volt (or eV) or million electron volt (MeV) units, 1 MeV being equivalent to 1.60×10^{-6} erg. Nuclear masses are generally expressed in atomic mass units (or amu), these being the masses on a scale in which the mass of the common isotope of carbon (^{12}C) is taken as precisely 12.0000. In terms of familiar mass units, 1 amu is 1.66×10^{-24} gram. Upon making the appropriate substitution into equation (1.1), it is found that

$$E(\text{MeV}) = 931 \, m \, (\text{amu}). \qquad (1.2)$$

Consequently, the energy in MeV accompanying a nuclear reaction is equal to the decrease in mass in amu multiplied by 931.

Fission and Fusion

1.02 Two types of nuclear reactions, in which there is a decrease of mass, are used for the large-scale release of energy in weapons. These are (*a*) fission, i.e., the splitting of a heavy nucleus into a pair of lighter ones, and (*b*) fusion, i.e., the combination of two very light nuclei to form a heavier one. The underlying reason why these processes are accompanied by a liberation of energy (and decrease in mass) is that in each case the total energy of attraction (or binding energy) among the constituent protons and neutrons, i.e., the nucleons, is smaller in the initial nucleus (or nuclei) than it is in the products of the reaction. It is a fundamental law of nature that the rearrangement of a system from a weakly bound state to a more tightly bound state must be accompanied by a release of energy.

Binding Energy

1.03 The magnitude of the net attractive energy, i.e., the binding energy, of the nucleons in any nucleus can be calculated from the masses of various particles. Consider an atomic species (or nuclide) of atomic number Z and mass number A, so that the nucleus of the atom contains Z protons and $A - Z$ neutrons. For electrical neutrality, the atom as a whole must have, in addition, Z extranuclear electrons. If m_p, m_n, and m_e are the masses of the proton, neutron, and electron, respectively, the sum of the masses of the constituents of the atom is then $Zm_p + Zm_e + (A - Z)m_n$. Suppose that the actual atomic mass, as determined by a mass spectrometer or in other ways, is M; then the mass defect (M.D.) of the particular isotope is defined by

$$\text{M.D.} = [Z(m_p + m_e) + (A - Z)m_n] - M$$
$$= Zm_H + (A - Z)m_n - M,$$

where $m_p + m_e$ has been replaced by m_H, the mass of the hydrogen atom, to which it is essentially equivalent.

1.04 From the arguments given above, it can be seen that the mass defect is a measure of the energy which would be released if the individual Z protons and $A - Z$ neutrons combined to form the given

nucleus.* This energy is numerically equal, but opposite in sign, to that which would have to be supplied to break up the nucleus into its constituent nucleons, i.e., the binding energy of the nucleus. Consequently, the mass defect of a nuclide can be related to the nuclear binding energy (B.E.) by utilizing equation (1.2); thus,

$$\text{B.E. (in MeV)} = 931 \times \text{M.D. (in amu).} \qquad (1.3)$$

Since the masses of the proton and neutron in amu are known and M can be determined, the M.D. can be derived from experimental data and from this the B.E. can be readily obtained for any nuclear species.

1.05 A useful quantity for practical purposes is the binding energy per nucleon, i.e., B.E./A, where A is the mass number which is equal to the number of nucleons in the nucleus. The values of B.E./A for many stable nuclides have been determined, from their known atomic masses; when plotted against the respective mass numbers, as in Fig. 1.1, the results are found to
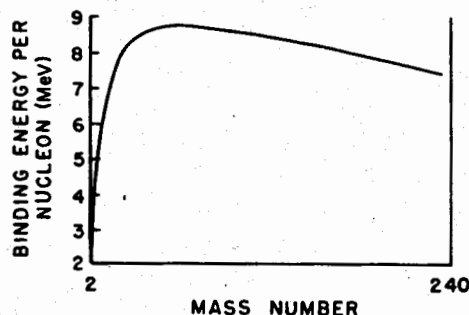

Figure 1.1

fall on, or very close to, a continuous curve. The significant aspect of this curve, for the present purpose, is that the mean binding energy per nucleon is less for the lightest and for the heaviest nuclei than it is for those of intermediate mass. It is this fact which accounts for the liberation of energy that accompanies either fission of heavy nuclei or fusion of light nuclei. In each case, the total binding energy in the initial nucleus (or nuclei) is less than that in the reaction products.

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*The Z electrons contribute a small amount of energy, but this is largely allowed for in the replacement of $m_p + m_e$ by m_H.

RELEASE OF FISSION ENERGY

Calculation of Fission Energy

1.06 The most accurate method for determining the energy released in fission is from the known masses (in amu) of the nucleus undergoing fission and of the nuclei formed in the process. A simple, although less exact, procedure is the following. It will be seen shortly that fission of uranium-235, for example, can result from the absorption of a neutron, and in the process two lighter fission-product nuclei and two or three free neutrons are formed. The uranium-235 nucleus contains 235 nucleons and so the fission-product nuclei will have a total of $235 + 1 - 2$ (or 3), i.e., 234 (or 233) nucleons, depending upon whether two or three neutrons are released; the latter number will be used for the present calculation. In uranium-235, the mean binding energy per nucleon (Fig. 1.1) is about 7.6 MeV; hence, if the 235 nucleons which make up the uranium-235 nucleus were combined, the energy released would be given by

$$235 \text{ Nucleons} \rightarrow \text{Uranium-235} + (235 \times 7.6) \text{ MeV.}$$

Nearly all the fission-product nuclei have mass numbers in the range from 95 to 140, and for such species Fig. 1.1 shows that the mean binding energy per nucleon is roughly 8.5 MeV. Consequently, the combination of the 233 nucleons to produce two fission-product nuclei can be represented by

$$233 \text{ Nucleons} \rightarrow \text{Two fission-product nuclei} + (233 \times 8.5) \text{ MeV.}$$

Upon subtracting the two energy expressions, it is seen that

$$\text{Uranium-235} \rightarrow \text{Fission product nuclei} + (233 \times 8.5) - (235 \times 7.6) \text{ MeV.}$$

The free neutrons absorbed and released in the fission process can be neglected in this calculation. Hence, the fission of a uranium-235 nucleus is accompanied by the release of $(233 \times 8.5) - (235 \times 7.6)$ MeV, i.e., about 200 MeV, of energy. The significance of this amount of energy in terms of more familiar units will be given below.

The Fission Process

1.07 The fission of heavy nuclei can be brought about in several different ways, but there is only one

that is of importance for the practical release of
nuclear energy. This is fission initiated by neutrons.
The reason is that the fission process is always
accompanied by the release of neutrons, which can
produce fission in other nuclei. Hence, once fission by
neutrons has been initiated in a quantity of material, a
chain reaction, carried on by neutrons, is possible with
the continuous release of energy. Only three nuclear
species, namely, uranium-233, uranium-235, and pluto-
nium-239, need be considered here for use in a fission
chain process for two reasons. First, although these
substances are radioactive and decay with the emission
of alpha particles, they have relatively long half-lives
and so are moderately stable.* All other fissile nuclides
have such short half-lives and decay so rapidly that
they have no practical value; in any event their strong
radioactivity would make them difficult to handle.
Second, of the relatively stable species, the three
mentioned above are the only ones which will undergo
fission as a result of the capture of neutrons of all
energies, e.g., from less than an electron volt (slow
neutrons) to millions of electron volts (fast neutrons).

1.08 The common isotope uranium-238, which
constitutes about 99.3 percent of the element in
nature, requires neutrons of about 1-MeV energy to
cause fission at an appreciable rate.† Most of the
neutrons produced in fission actually have higher
energies, but they lose energy rapidly in (inelastic)
collisions, so that they are brought below the threshold
of about 1 MeV for significant fission of uranium-238.
Consequently, the maintenance of a chain reaction in
uranium-238 is impossible. Nevertheless, fission of this
isotope by fast neutrons does take place and the energy
released can make a significant contribution to the
total energy produced in nuclear weapons.

1.09 Of the three fissile species, only uranium-235
is found in nature; the other two are produced by
artificial nuclear reactions (§1.53). Furthermore, only
two, namely, uranium-235 and plutonium-239, are
being used in weapons. Although uranium-233 has
fission characteristics which would appear to make it
of interest for use in certain types of artillery shells of
small caliber, the strong gamma-ray activity of asso-

ciated products introduces serious fabrication and
handling problems.

1.10 The fission process may be regarded as
yielding three types of products: (a) lighter nuclei,
called fission fragments, (b) neutrons, referred to as
fission neutrons, and (c) energy. Thus, taking uranium-
235 as typical, the fission act may be represented by

Uranium-235 + n → two fission fragments

+ 2 to 3 n + energy,
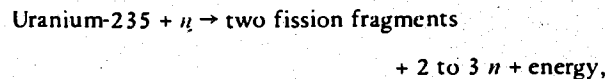
where n indicates a neutron. These products will be
considered in turn. It may be mentioned that the
general discussion of the fission process given below is
applicable to all three fissile substances. Such differ-
ences in behavior as do exist arise from differences in
the average number of neutrons produced when fission
occurs and in the relative probabilities (or cross
sections) of fission and nonfission reactions for neu-
trons of a given energy.

Fission Products

1.11 Uranium-235 (and other) nuclei can fission in
a number of different ways, leading to several different
pairs of fission fragments. Most of these are radio-
active, decaying by the emission of beta particles and
frequently also gamma rays. On the average, each
radioactive fission fragment undergoes three or four
stages of beta decay before a stable species is formed.
As a result, a complex mixture is produced consisting
of more than 400 different nuclides, including isomeric
states, of nearly 40 different elements. This complex
system of fission fragments and their decay products is
referred to by the general name of fission products.
The mixture contains many radioisotopes and their
half-lives range from a second or so to more than a
million years. It is the fission product mixture that
contributes nearly all of the radioactivity of the fallout
produced by a nuclear explosion in the air. If the
fireball touches the surface of the ground or of the
ocean, radioactivity induced by neutrons in the earth
or seawater is also important.

Fission Neutrons

1.12 The number of neutrons released when a
nucleus undergoes fission varies somewhat with the
particular mode of fission, but the *average* number of
fission neutrons is well defined. The value depends on
the energy of the neutrons which cause fission, and
increases to some extent with increasing neutron

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*The half-lives are as follows: uranium-233, 1.6 x 10^5
years; uranium-235, 7.1 x 10^8 years; plutonium-239, 2.4 x 10^4
years.

†A distinction is sometimes made between the "fissile"
nuclei, such as uranium-235, etc., in which fission can be
produced by a neutron regardless of its energy, and the
"fissionable" nuclei, such as uranium-238 and thorium-232,
which require neutrons of high energy to cause fission.

energy. The results of estimates based on experimental measurements are given in Table 1.1 of the average

Table 1.1 NEUTRONS RELEASED PER FISSION

Nuclide	Neutron Energy		
	~0 MeV	0.5 MeV	14 MeV
Uranium-235	2.43	2.49	4.1
Plutonium-239	2.80	2.85	4.9
Uranium-233	2.45	2.51	4.2

number, ν, of neutrons produced per fission caused by (a) slow neutrons, i.e., neutrons of essentially zero energy, (b) neutrons of about 0.5-MeV energy, which is roughly the average energy of the neutrons which maintain the chain in a simple fission weapon, and (c) neutrons of 14-MeV energy which contribute greatly to the fission energy of boosted and many two-stage devices (Chapters 6 and 7).

1.13 The neutrons emitted in fission fall into two categories, namely, prompt neutrons and delayed neutrons. The former are all released within something like 10^{-14} sec of the fission process, but the latter continue to be emitted for a few minutes. For uranium-235 fission, the prompt neutrons constitute 99.35 percent of the total fission neutrons and for plutonium-239 they represent about 99.75 percent. Because the time scale in nuclear explosions is very short, delayed neutrons play essentially no part in the fission chain reaction. In reactors for the controlled release of nuclear energy, however, these neutrons are of great significance.

Fission Energy

1.14 The rough estimate made earlier indicated that about 200 MeV of energy are produced per nucleus undergoing fission. More precise calculations, based on nuclear masses, and experimental measurements have shown that this is a good approximation for both uranium-235 and plutonium-239. The atomic mass in grams, i.e., 235 grams of uranium-235, contains 6.02×10^{23} nuclei, and the complete fission of this amount of uranium-235 would yield $6.02 \times 10^{23} \times 200 = 1.20 \times 10^{26}$ MeV or 1.93×10^{20} erg, since 1 MeV is equal to 1.60×10^{-6} erg. Making use of the fact that 1 calorie is equivalent to 4.18×10^{7} ergs, it can be readily shown that complete fission of all the nuclei in 1 kilogram of fissile material

would result in a total energy release of 2.0×10^{13} calories.

1.15 Only part of the energy of fission is immediately available in a nuclear explosion, since most of the radioactive decay energy of the fission products is released over a long period of time. It is usually accepted that about 90 percent of the fission energy contributes to the explosion, so that in a weapon the fission of 1 kilogram of material would produce explosive energy of about 1.8×10^{13} calories. The energy liberated in the explosion of 1 ton of TNT is taken to be 10^{9} calories (or 4.18×10^{16} ergs), and so 1 kilogram of fissile (or fissionable) material is equivalent in explosive power to 18,000 tons, i.e., 18 kilotons (or 18 kt) of TNT. From these results it is readily found that complete fission of 0.056 kg (or 56 g) or of 1.45×10^{23} nuclei of fissile material produces the equivalent of 1 kt of TNT of explosive energy. In other words, the energy per fission is about 7.0×10^{-24} kt TNT equivalent.* In stating the energy yields (or, in brief, the yields) of nuclear weapons, the basic unit, for very low yields, is the ton, with the kiloton (or 1,000 tons), i.e., 1 kt, and the megaton (or 1,000,000 tons), i.e., 1 Mt, of TNT equivalent being used for higher yields.

THE FISSION CHAIN REACTION

Condition for Chain Reaction: Critical Size

1.16 The condition for a self-sustaining fission chain reaction is that, on the average, the neutrons released in one act of fission shall cause (at least) one subsequent fission. Since the average number of neutrons produced in an act of fission is greater than two (see Table 1.1), it would appear, at first sight, that a chain reaction in uranium-235 or plutonium-239 would be inevitable. However, this is not so, because an appreciable proportion of the neutrons produced are lost in various ways. In a nuclear fission weapon, for example, an important source of loss is by leakage (or escape) of neutrons from the reacting material. Thus, many neutrons avoid being captured by a fissile nucleus by escaping from the system entirely. Some neutrons are also lost by parasitic capture, i.e., by capture in nonfission reactions of various kinds, either by the fissile species itself or by other nuclides which may be present.

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*The energy per fission of uranium-235 is currently taken as 6.793×10^{-24} kt, i.e., 17.4 kt/kg; for plutonium-239 the values are 7.030×10^{-24} kt per fission and 17.7 kt/kg, respectively.

1.17 The fraction of neutrons escaping from a system in which fission is occurring can be decreased by increasing the mass, i.e., by increasing the size at constant density of fissile material.* Since neutrons are produced by fission throughout the whole volume, whereas loss by escape takes place only from the exterior surface, it is evident that the escape probability will decrease as the volume-to-area ratio of the system is increased. This can be achieved, for a given geometry (shape), by increasing the dimensions of the fissile material at constant density.

1.18 In a very small mass of fissile material a self-sustaining chain reaction will not be possible under normal circumstances because of the large proportion of neutrons that escape. But as the size is increased (at constant density), the fraction of fission neutrons lost will decrease and ultimately a point is reached when one neutron will be available to carry on the fission chain for every neutron causing fission. The system is then said to be critical, and a self-sustaining chain reaction is just possible. If a system is smaller than the critical size (or mass), it is referred to as subcritical, and if larger, it is supercritical. In the latter case there are more neutrons available for fission at the end of any generation than were captured in fission reactions at the beginning of that generation. It will be seen shortly that the critical size (or mass) is dependent on the nature of the fissile material, its shape, and several other factors. For a given set of conditions, however, the critical size (or mass) has a definite value which can be determined by experiment or can sometimes be calculated.

Convergent, Stationary, and Divergent Chains

1.19 If ν is the average number of neutrons produced in each act of fission, for the existing neutron energy distribution (or neutron spectrum), and l is the number lost by escape and in other ways, e.g., by nonfission capture, then $\nu - l$ is the number of neutrons which can cause further fission; let this be represented by k, called the effective multiplication factor, i.e.,

$$k = \nu - l. \qquad (1.4)$$

Thus, for every neutron causing fission in one generation, k neutrons will cause fission in the next generation. Alternatively, k may be defined as the ratio of the number of neutrons in any one generation to the

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*The reason for specifying constant density of fissile material will be apparent later (§ 1.40).

number in the preceding generation. Hence, in accordance with the statements made above, k is less than unity for a subcritical system; in a critical system $k = 1$; and in a supercritical system k is greater than unity.

1.20 Suppose S neutrons are introduced into a mass of fissile material and cause fissions to occur; then kS neutrons will be present in the next generation, $k^2 S$ in the third, and so on. Since, for a subcritical system, $k < 1$, it is evident that as g, the number of generations, increases, $k^g S$ will approach zero. In other words, the number of neutrons present will gradually decrease from one generation to the next, because more neutrons are lost in various ways than are being produced by fission. There is, consequently, a convergent or decaying chain which gradually dies out. For a system of critical size, $k = 1$ and then $k^g S$ is always equal to S. The number of neutrons thus remains constant from generation to generation; this is referred to as a stationary chain. Finally, if $k > 1$, as is the case for a supercritical system, $k^g S$ increases steadily. The fission chain is then said to be a divergent or expanding chain.

1.21 In the foregoing discussion no distinction has been made between the prompt and delayed fission neutrons. As stated earlier, however, it is essentially the prompt neutrons only which are significant in fission weapons. The effective critical size (or mass) of a weapon is thus determined by the availability of the prompt neutrons, without regard to those which are delayed. This situation is often described as prompt critical. Since more than 99 percent of fission neutrons are prompt, the neglect of the delayed neutrons has little effect on the value of ν. In other words, the data in Table 1.1 may be taken as being approximately applicable to weapons. Critical masses determined by experiment (§ 1.29) include the contribution of delayed neutrons, so that the prompt critical mass applicable to weapons is very slightly larger than the value measured in the laboratory. Again, the difference is small, although corrections can be applied if necessary.

Factors Affecting Critical Mass

1.22 The critical size (or mass) of a given fissile material depends on a number of factors, as mentioned earlier. For example, the shape (or geometry) of the system has an influence on criticality because of the variation in the ratio of volume to surface area. As seen in § 1.17, this ratio determines the fraction of neutrons lost by leakage from the system. The optimum condition of minimum critical mass is obtained for a
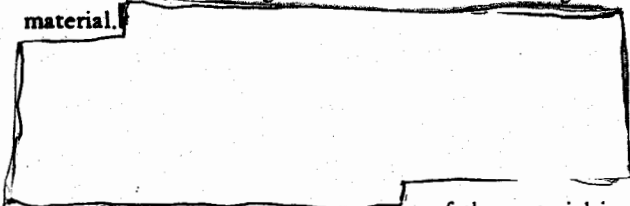
sphere; this has the largest ratio of volume to area, so that the fraction of neutrons produced which escape from the system is less than for any other shape of the same mass. Thus, the critical mass of a sphere is less than for any other geometrical form of the given material.

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1.23 As far as the composition of the material is concerned, it is evident that the presence of impurities will increase the critical mass of the fissile species. Apart from the fact that the impurity may cause a loss of neutrons by parasitic capture, it adds area to the system, which increases the probability of neutron escape, without the compensating production of neutrons by fission in the increased volume. Similar considerations apply to the situation in which a given fissile substance, e.g., plutonium-239, exists in two or more allotropic forms with different densities. The less dense delta form of plutonium metal has a higher critical mass than the more dense alpha form because of the larger surface area for a given mass. The rate of neutron production by fission depends on the mass, i.e., the number of fissile nuclei, but the rate of loss by escape is greater for the delta-plutonium since the area is larger for the same mass.

1.24 The value of ν also affects the critical size. The larger the magnitude of ν, the smaller will be the critical size (or mass) under the same general conditions. If more neutrons are formed per fission, it will be possible to tolerate a somewhat larger loss and yet attain criticality. This is apparent from the fact that the critical condition is that $k = \nu - l = 1$, so that an increase in l can be compensated by an increase in ν. It is because both ν and the fast-neutron fission cross section of plutonium-239 are larger than for uranium-235 that the critical size of the former is less than that of the latter, under equivalent conditions. Since the densities of plutonium and uranium are not greatly different, the same relationship applies to the respective critical masses (see Table 1.2).

1.25 For a specified shape and composition, the size of a critical system can be decreased by surrounding it with a material which scatters some of the escaping neutrons back into the fissile core. By reducing the fraction of neutrons which escape completely, a smaller size (or mass) can become critical (§1.34). Such a scattering material, on account of its

function, is sometimes referred to as a neutron reflector or as a neutron tamper.

1.26 In nuclear weapons, the fissile material is surrounded by a tamper or, more specifically, an inertial tamper, the mass of which delays expansion of the exploding material and permits a higher energy yield to be obtained from the system undergoing fission. This inertial tamper also serves as a neutron reflector or neutronic tamper. In some cases, however, the neutronic aspect is more important than the inertial character of the tamper.

1.27 As is to be expected, increasing the thickness of the tamper decreases the escape of neutrons and thus makes possible a smaller critical mass of the core of fissile material. However, it has been shown by calculations and verified experimentally that when the neutronic tamper thickness reaches a certain value, there is little more to be gained by a further increase of thickness (Fig. 1.2). Thus, when the thickness is about two neutron scattering mean free paths, the effectiveness in decreasing the critical mass is within a few percent of that for an infinitely thick tamper.* In natural uranium, which was at one time used as a
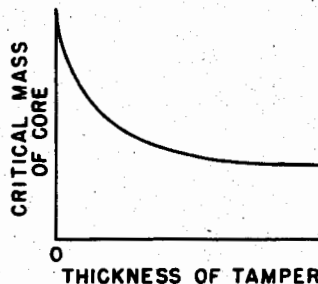
Figure 1.2

tamper, the scattering mean free path of fast (1 MeV) neutrons is about 4 cm, i.e., 1.6 in., in metal of normal density. The value is proportionately less in compressed uranium of higher density. In weapons of low mass, beryllium is a common tamper material; the scattering mean free path is somewhat longer than in uranium because of the lower density.

1.28 Another important factor which affects the critical size is the energy (or speed) of the neutrons causing fission. For several reasons, some of which will be explained later, nuclear weapons are designed so

*The scattering mean free path is the total distance a neutron travels before undergoing a scattering collision with a nucleus in the given tamper material.

...at the fission chain is maintained by fast neutrons, with energies in the range of approximately 0.1 to 2 MeV. In the subsequent treatment it will be assumed, therefore, that fast-neutron fission makes the main contribution to the chain reaction.

Determination of Critical Mass

1.29 Critical masses can now be calculated with a considerable degree of accuracy, provided all the conditions are known exactly. It is desirable, however, to check these values by experimental measurements. Because of the danger involved in handling critical assemblies, the general procedure is to extrapolate from observations made on a number of subcritical systems of increasing mass.

1.30 It was seen in § 1.20 that the introduction of S neutrons into a fissile material results in the presence of kS neutrons in the first generation, k^2S in the second, and so on. If a steady source emitting S neutrons per second is used, then all generations will be present in the system, so that the total rate of neutron production is $S + kS + k^2S + ...$. The ratio of the rate of neutron formation to the source strength is called the neutron multiplication, M; thus,

$$M = \frac{S + kS + k^2S + ...}{S} = 1 + k + k^2 + k^3 + ...$$

For a subcritical system, k is less than unity; the series $1 + k + k^2 + ...$ is then convergent and for a large number of generations it is equal to $1/(1 - k)$. Consequently,

$$M = \frac{1}{1 - k}. \tag{1.5}$$

Since $1 - k$ is finite and positive, M has a finite value for a subcritical system. For a critical system, however, $k = 1$; $1 - k$ is then zero and M becomes infinite.

1.31 For the determination of critical masses, it is more convenient to consider the reciprocal of the multiplication, i.e., $1/M$; by equation (1.5), this is given by

$$\frac{1}{M} = 1 - k.$$

Provided the system is subcritical, $1/M$ is finite but less than one, but for criticality $k = 1$ and so $1/M$ is zero. The neutron multiplication is observed for several subcritical systems containing different masses of fissile material, and the experimental values of $1/M$ are plotted against the masses. The extrapolated mass corresponding to $1/M = 0$ is then the critical mass under the existing conditions.

1.32 The neutron multiplication is determined by placing a steady neutron source inside an assembly of active material of prescribed shape and composition and of known mass. The measured rate of arrival of neutrons at a counter located outside the assembly is proportional to $S + kS + k^2S + ...$. The rate of arrival from the same source in the absence of fissile material is proportional to S. Hence, the ratio of the measurements with and without the assembly of active material is equal to the multiplication M. Determinations of M are made in this manner, with the same source and detector location, for a number of assemblies of increasing mass, and $1/M$ is plotted against the mass, as in Fig. 1.3. The extrapolated mass for $1/M = 0$ is the critical mass. By changing the position of the detector, the apparent multiplication is changed, but the $1/M$ values should always extrapolate to the same point, as shown by the two curves in the figure.

1.33 The procedure described above can be used to determine critical masses for both tamped (reflected) or untamped assemblies. It can also be applied to actual mockups of fission weapons. Some of the

MASS OF FISSILE MATERIAL
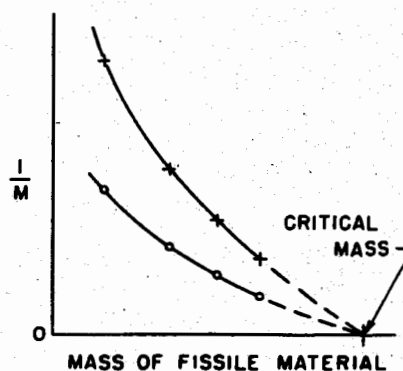
Figure 1.3

components, e.g., fissile substances, are the same as would be used in the weapon, whereas others, e.g., high explosive, are simulated by other materials with similar neutronic properties.

1.34 Under precisely specified conditions, and for a given core material, there is a definite mass that is just critical. The critical masses of spheres of metallic uranium-235 (93.2 weight percent enrichment), of the alpha- and delta-phases of plutonium-239,

for fast-neutron fission are given in Table 1.2. Values are quoted for bare, i.e., untamped spheres, as well as for spheres with tampers of

The name gun-type originates from the fact that in devices of this kind the two pieces of fissile material are located near opposite ends of a gun barrel.*

Table 1.2 CRITICAL MASSES OF SPHERES

Fissile Material	^{240}Pu (wt %)	Density (g/cm^3)	Critical mass (kg) Bare	U	U	Be	Be
Uranium-235 (93.2 wt %)	-	18.8	52.5				
Plutonium-239 (α)	4.8	19.7	10.5				
Plutonium-239 (δ)*	4.8	15.8	16.6				
Plutonium-239 (δ)*	21	15.8	18.8				

*Contains ~1 wt % gallium

beryllium, respectively. The effect of the neutronic tamper in reducing the critical mass is very striking. It will be noted, too, in accordance with remarks made earlier, that the critical mass of delta-plutonium is larger than that of the alpha form, whereas both are smaller than the critical mass of uranium-235.

FISSION WEAPONS

Gun Method of Assembly

1.35 As long as a mass of fissile material is less than the critical value for the existing conditions, that is to say, provided the system is subcritical, there is no danger of a divergent, or even a stationary, chain reaction. But, if energy is to be released in a nuclear explosion, the system must be made critical and, in fact, highly supercritical, as will be seen shortly. There are two general ways utilized in weapons whereby a subcritical system of fissile material is rapidly converted into one that is supercritical.

1.36 The first is generally referred to as the gun method of assembly. Two portions of material of subcritical size are brought together very rapidly, for reasons given below, so that the combined mass is supercritical. If a burst of neutrons is then introduced, a divergent fission chain is initiated and a rapid release of energy occurs in a very short time. This is the principle used in the so-called "gun-type" weapons:

1.37 Although the gun assembly method for attaining criticality is satisfactory when uranium-235 is the fissile material, it has a serious drawback when plutonium-239 is used. This arises from the presence of the higher isotope, plutonium-240. Because of the way it is produced (§1.53), plutonium-239 is invariably associated with a certain proportion—generally from about 2 to 7 percent—of the higher isotope, plutonium-240. The latter happens to have a high probability for undergoing spontaneous fission, i.e., without the intervention of neutrons. The spontaneous fission rate of plutonium-240 is, in fact, about 440 fissions per second per gram. Since more than two neutrons are liberated per fission, on the average, this means that 1 gram of plutonium-240 emits over 1000 neutrons per second.

1.38 An efficient use of the fissile material in a simple gun-type weapon requires that the chain reaction be initiated by neutrons only when the assembly has attained its maximum criticality. the considerable neutron background, arising from the presence of plutonium-240, may result in initiation of a self-sustaining chain reaction as soon as the assembly becomes just critical. If this occurs, there will be little

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*It is purely a coincidence that in most nuclear artillery shells the gun type of assembly is used. There is, however, another type of artillery shell which does not employ gun assembly (§4.23).

or no explosion, since the neutron density will not increase rapidly and energy, resulting from fission, will not be produced fast enough; the reasons for such behavior will be apparent later. On the other hand, uranium-235 has a small neutron background and therefore can be used in a gun-type weapon. If assembled rapidly enough, there is little probability of premature initiation, or "preinitiation," as it is called, immediately upon the system becoming critical. When maximum supercriticality is attained, neutrons are introduced deliberately from a suitable source to initiate the fission chain reaction. In this way, the optimum efficiency can be realized in the use of the fissile material.

Compression Method (Implosion Weapon)

1.39 Because of the probability of preinitiation, and low efficiency, of a gun-assembly weapon using plutonium-239, an alternative method for attaining criticality (or supercriticality) was developed, based on the compression of the subcritical fissile mass. This procedure turned out to be so successful and gave so much better efficiency that the gun type of assembly has been utilized only in a relatively few weapons for special purposes, e.g., in artillery-fired shells and in rugged, impact-resistant bombs designed to penetrate some distance into the ground before exploding. Apart from these particular cases, the compression method is invariably used to attain supercriticality in fission weapons.

1.40 The principle of the method is that if a mass of fissile material is compressed, the rate of production of neutrons by fission in the subcritical state is essentially unchanged, since it depends mainly on the number of nuclei present. Actually, there will be some increase in the neutron production in convergent chains. On the other hand, the number of neutrons lost by escape is decreased as a result of compression because of the smaller surface area of the given mass. Consequently, a quantity of material which is subcritical in the normal state can become supercritical when compressed. The introduction of neutrons to initiate the fission chain at (or close to) the time of maximum compression—and, hence, of maximum supercriticality—results in an efficient use of the fissile material in causing an explosion.

1.41 In practice, the compression occurs very rapidly.

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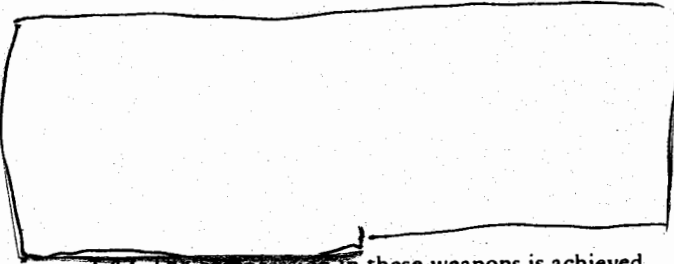
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1.42 The compression in these weapons is achieved by the use of a powerful conventional (chemical) high explosive which surrounds the core of fissile material. By the use of explosive charges of special design much of the energy of the explosion is directed inward, thereby causing the material in the interior to be compressed in a spherically symmetric manner. It is for this reason that the term "implosion" is applied to weapons of this type.

1.43 An approximate derivation of the relationship between the degree of compression and the critical mass of fissile material is the following.[†] The total mean free path of a neutron is the average distance it travels before it interacts in any way with a nucleus. The proportion of neutrons which do not interact but escape from the system may be expected to be determined by the ratio of the dimensions, e.g., the radius of a sphere, to the mean free path. It may be concluded, therefore, that for a given fissile (core) material, under specified conditions, the critical radius should be approximately proportional to the neutron mean free path; thus, if R_c is the critical radius and λ is the mean free path in the material,

$$R_c \propto \lambda. \qquad (1.6)$$

1.44 The greater the probability of the interaction of a neutron with a nucleus, the smaller will be the distance the neutron travels before interacting. Hence, the neutron mean free path is related inversely to its interaction probability. This probability is proportional to the number of fissile nuclei per unit volume, and hence to the density; if ρ is the density of the core material, then

$$\lambda \propto \frac{1}{\rho}. \qquad (1.7)$$

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[†]The purpose of the discussion in §1.43 through §1.47 is to provide a general basis for understanding the effect of compression on criticality. It is not intended to imply that the methods are currently used. At present, computer calculations, which can take many variables into consideration, are employed to derive criticality conditions. The term "crit" in §1.46 is now more or less obsolete in weapons calculations.

It follows therefore from equations (1.6) and (1.7) that

$$R_c \propto \frac{1}{\rho}. \qquad (1.8)$$

1.45 The critical mass, M_c, is equal to the product of the critical volume, which is $\frac{4}{3}\pi R_c^3$, and the density of the fissile material; hence,

$$M_c = \frac{4}{3}\pi R_c^3 \rho .$$

Upon substituting equation (1.8) for R_c, it is seen that

$$M_c \propto \frac{1}{\rho^2}. \qquad (1.9)$$

The density of the material is dependent upon the degree of compression; thus, if η is the compression ratio, i.e., the ratio of the volume before to that after compression, then

$$\rho \propto \eta,$$

and substitution in equation (1.9) leads to the result

$$M_c \propto \frac{1}{\eta^2}. \qquad (1.10)$$

The critical mass of a given fissile material, under specified conditions, is thus inversely proportional to the square of the compression ratio. The proportionality constant is readily derived by writing M_{c0} for the critical mass of the uncompressed material, i.e., when $\eta = 1$. It follows then from equation (1.10) that

$$M_c = \frac{M_{c0}}{\eta^2}. \qquad (1.11)$$

1.46 An alternative way of stating this result is in terms of the number of "crits" (or critical masses), C, present in the compressed core. If M is the actual mass of fissile material, the number of crits in the compressed state is defined by

$$C = \frac{M}{M_c}.$$

Combination with equation (1.11) then yields

$$C = \frac{M\eta^2}{M_{c0}} = C_0\eta^2, \qquad (1.12)$$

where M/M_{c0} has been replaced by C_0, the number of crits before compression.

The introduction of neutrons into this highly supercritical system resulting from compression will cause a very rapidly divergent fission chain reaction to develop. In these circumstances there is very efficient use of the fissile material for the release of energy. It is the high degree of supercriticality (and increased efficiency) attainable by compression that constitutes the great advantage of implosion-type weapons over those of the gun type.

1.47 Strictly speaking, the relationship of the number of crits to the square of the compression holds only for a bare core. For a tamped core, a more correct form of equation (1.12) is
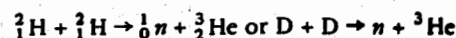
$$C = C_0\eta_c^{1.2}\eta_t^{0.8},$$

where η_c is the compression of the core and η_t is that of the tamper. Since the tamper is generally compressed less than the core, a good approximation for weapons is to write

$$C = C_0\eta_c^{1.7}.$$

The effect of compression is still substantial, although not as large as is implied by equation (1.12).

Fusion Reactions in Fission Weapons

1.48 It was stated in §1.02 that the large-scale release of energy in weapons is possible by making use of fusion reactions in which two very light nuclei combine (or fuse) together to form a nucleus of larger mass. However, apart from the application of nuclear fusion reactions as a source of energy, described in Chapter 7, certain fusion processes are important in the design of fission weapons for another reason. The significance of these reactions does not lie in the energy released but in the neutrons which are produced. Three fusion reactions, involving the less common isotopes of hydrogen, namely, deuterium ($^2_1$H or D) and tritium ($^3_1$H or T), are of interest in this connection. Two of these reactions are between pairs of deuterium nuclei (deuterons) only, i.e.,

$$^2_1H + {}^2_1H \rightarrow {}^1_0n + {}^3_2He \text{ or } D + D \rightarrow n + {}^3He$$

and

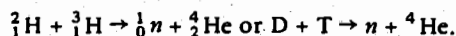$$^2_1H + ^2_1H \rightarrow ^1_1H + ^3_1H \text{ or } D + D \rightarrow H + T,$$
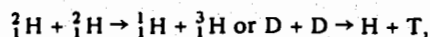
which take place at about the same rate, and the third is a much more rapid interaction between a deuteron and a tritium nucleus (triton), i.e.,

$$^2_1H + ^3_1H \rightarrow ^1_0n + ^4_2He \text{ or } D + T \rightarrow n + ^4He.$$

1.49 In a mixture of deuterium and tritium, the main reaction is between a deuteron and a triton which results in the formation of a neutron as one of the products. In deuterium alone, a neutron is also a product in one of the deuteron-deuteron (D-D) reactions. In the other D-D reaction, a triton is formed and this readily reacts with a deuteron to produce another neutron. Both deuteron-deuteron (D-D) and deuteron-triton (D-T) reactions are employed to provide neutrons for initiating fission chains. In addition, the high-energy (14 MeV) neutrons liberated in the D-T reaction are used in many fission weapons to achieve what is known as "boosting." Neutrons from the D-T reaction are introduced at a late stage of the fission chain in order to maintain and enhance the progress of the fission reactions. There is a considerable increase in the energy released because of the greatly improved efficiency in utilization of the fissile material. The energy contributed under the circumstances by the D-T fusion reaction is, however, quite small in comparison with that from fission.

PRODUCTION OF WEAPONS MATERIALS

Uranium-235

1.50 The two important fissile materials, namely, uranium-235 and plutonium-239, are both produced from natural uranium but by entirely different procedures. Ordinary uranium contains about 0.7 percent of uranium-235, together with about 99.3 percent of uranium-238 and a trace (0.006 percent) of uranium-234. The proportion of uranium-235 is increased by a process involving diffusion or, more correctly, effusion through porous barriers of the vapor of uranium hexafluoride (UF_6) made from natural uranium. The hexafluoride of the lighter isotope diffuses more rapidly than does that of the heavier species, and by the use of several thousand diffusion stages enrichments of over 90 percent are obtained, i.e., the material produced contains over 90 percent of uranium-235. The most common product for weapons use

consists of about 93.2 weight percent uranium-235, the remainder being mainly uranium-238 and a small proportion of uranium-234. This product is commonly known as "oralloy," the two initial letters standing for Oak Ridge, where the material was first made in quantity.*

DOE b(3)

1.51 The highly enriched uranium hexafluoride obtained from the gaseous diffusion plant is converted into the tetrafluoride (UF_4) by reduction with hydrogen (mixed with some fluorine). The tetrafluoride, which is a solid with a high melting point (close to 1000°C), is mixed with calcium and heated in a closed steel vessel lined internally with a refractory material. The calcium reduces the uranium tetrafluoride to uranium metal which is separated from the slag of calcium fluoride. Volatile impurities are removed by heating the liquid metal in a vacuum and the resulting product is of a high degree of purity.

1.52 The residual material from the isotope separation (gaseous diffusion) plant consists of uranium hexafluoride which has been depleted in uranium-235. In other words, it contains more than the normal 99.3 percent of uranium-238. This is converted into uranium metal by a procedure similar to that described above. It is referred to as depleted uranium or, in the weapons program, as D-38, because at one time it contained 0.38 percent uranium-235, although greater depletion has been regularly achieved. At one time it was called Q-metal, but this name is not now in common use.

Plutonium-239

1.53 The element plutonium does not occur in nature, except in insignificant traces. Consequently, the plutonium-239 used in weapons is obtained artificially by a series of nuclear reactions resulting from exposure of uranium-238 to slow neutrons in a nuclear reactor. A nuclear reactor is a device in which a fission chain reaction is taking place in a controlled manner, as against the deliberately uncontrolled chain reaction in a weapon. If a material of low mass number, called a

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*The "alloy" part of the name originated from the designation Tube Alloys Limited applied to the British wartime atomic energy project. Natural uranium metal was thus called "tuballoy," a term still in common use, and then oralloy was adopted for the material highly enriched in uranium-235.

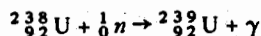moderator, is present, in addition to fissile material, the fission neutrons are slowed down. Such a nuclear reactor is thus a good source of slow neutrons.

1.54 Uranium-238 nuclei capture slow neutrons quite readily to form a higher isotope, uranium-239, with the emission of gamma radiation; thus, the (n,γ) reaction

$$^{238}_{92}U + ^1_0n \rightarrow ^{239}_{92}U + \gamma$$

occurs, where the subscripts give the atomic numbers and the superscripts the mass numbers in each case; the neutron (^1_0n) has a charge (atomic number) of zero and a mass of unity. The uranium-239 produced by neutron capture is radioactive, with a half-life of 23 min, emitting a beta particle. Representing the latter by $^0_{-1}\beta$, since it carries a unit negative charge and has essentially no mass, the radioactive decay process may be written as

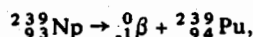$$^{239}_{92}U \rightarrow ^0_{-1}\beta + ^{239}_{93}Np,$$
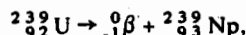
the product being an isotope of mass number 239 of an element of atomic number 93. This element, called neptunium (Np), is virtually nonexistent in nature.

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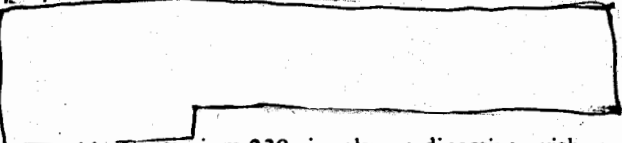
1.55 Neptunium-239 is also radioactive with a half-life of 2.3 days; it emits a beta particle according to the reaction

$$^{239}_{93}Np \rightarrow ^0_{-1}\beta + ^{239}_{94}Pu,$$

so that the product has a mass number of 239 and an atomic number of 94. The name plutonium, symbol Pu, has been given to the element with this atomic number. The isotope plutonium-239 is an alpha-particle emitter with a fairly long half-life—about 24,000 years—so that it is relatively stable. It may be mentioned that the decay product of plutonium-239 is the fissile uranium-235, which has a half-life of about 7×10^8 years. Hence, as far as fission is concerned, plutonium-239 could be stored for thousands of years with only minor deterioration. The little deterioration which does occur arises mainly from the fact that the average number of neutrons, ν, produced by fission of uranium-235 is somewhat less than that from plutonium-239 (§1.12).

1.56 For the production of plutonium-239, natural uranium metal is used as the fuel material in a nuclear reactor with graphite (Hanford) or heavy water (Savannah River) as the moderator. The uranium-235 in the fuel sustains the fission chain reaction and produces neutrons, some of which are captured by the uranium-238 with the consequent formation of plutonium-239, as described above. After being in the reactor for an appropriate time, the "spent" fuel is removed, dissolved in nitric acid, and the plutonium is extracted from the solution by the use of certain organic solvents. It is then reextracted into a water medium to give an aqueous solution of the nitrate, from which the plutonium is precipitated either as the peroxide or the oxalate. The solid compound is separated and heated with a mixture of hydrogen fluoride gas and oxygen to obtain plutonium tetrafluoride (PuF_4). The latter is finally reduced to plutonium metal with calcium (plus iodine).

1.57 Metallic plutonium exists in six different allotropic forms between room temperature and the melting point (641°C). The temperature ranges over which the various forms (or phases) are stable are shown in Table 1.3. It is seen that alpha-plutonium,

Table 1.3 PROPERTIES OF SOLID PHASES OF PLUTONIUM

Phase	Stability range (°C)	Density (g/cm³)
Alpha (α)	Below 115	19.86 (25°C)
Beta (β)	115–~200	17.70 (190°)
Gamma (γ)	~200–310	17.14 (235°)
Delta (δ)	310–458	15.92 (320°)
Delta prime (δ')	458–480	16.0 (465°)
Epsilon (ϵ)	480–641 (m.p.)	16.51 (490°)

referred to in §§1.23 and 1.34, which has a density of about 19.8 g/cm^3, is stable at ordinary temperatures. The high density is advantageous from certain weapon standpoints, since it permits attainment of criticality in a smaller mass than is possible with other forms of plutonium (cf. Table 1.2). On the other hand, alpha-plutonium is brittle and difficult to fabricate. Furthermore, the presence of certain impurities tends to stabilize the beta or delta phases to some extent,

DOE b(3)

parts made of nominal alpha-plutonium are not dimensionally stable. However, if these impurities are avoided, the problem of dimensional instability does not arise at ambient temperatures.

1.58 Although delta-plutonium is normally stable in the temperature range of 310 to 458°C, the addition

of 1 weight percent of gallium to plutonium stabilizes the delta phase at ordinary temperatures.

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Most of the plutonium used in stockpile weapons is in the stabilized delta phase. The material is much less brittle and easier to fabricate than alpha-plutonium; in fact delta-plutonium is said to resemble copper in this respect whereas the alpha-phase is more like cast iron. Delta-plutonium was used in the earliest implosion weapons because it could be fabricated more readily. It offered the additional advantage that its low density permitted the use of a larger mass of subcritical fissile material (Table 1.2), thereby making possible an increase in the total energy yield. This latter aspect of weapons design is, however, no longer significant.

1.59 In order to minimize the critical mass and thus the overall weight of a weapon, it would be desirable to use plutonium of high density. Alpha plutonium has consequently been employed to some extent, but, in addition to the fabrication difficulty already referred to, it is more sensitive to the action of neutrons (§ 8.12) than is the delta form.

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Plutonium-240 and -241

1.61 As the plutonium-239 accumulates in the reactor in which it is produced, it also captures neutrons, the rate of capture being proportional to the neutron density (or flux) in the reactor and to the concentration of plutonium-239 nuclei. The reaction which takes place is

$$^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{240}_{94}Pu + \gamma,$$

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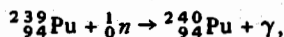
the product being plutonium-240, an alpha emitter of about 6600 years half-life. Because chemical processes are used for the extraction of plutonium from the spent reactor fuel, the two isotopes of this element are not separated from each other. Hence, plutonium-239 is always associated with a certain proportion of plutonium-240. Some of the latter is converted into plutonium-241 by neutron capture, but the amount of plutonium-240 (and plutonium-241) increases with both exposure time and the neutron density (or flux) in the reactor. In a production reactor, the practical limiting proportion of plutonium-240 is about 8 percent (~1 percent plutonium-241); it could go higher than this, as indeed it does in power reactors, but there would be a corresponding (and undesirable) loss of plutonium-239.

1.62 In a simple (unboosted) implosion type weapon, it is desirable that the plutonium-240 content be as small as possible for two reasons: (a) the high spontaneous fission rate of plutonium-240 causes a large neutron background (§ 1.37) which can result in the initiation of a fission chain before the optimum time, and (b) the fission of plutonium-240 requires neutrons of high energy and, in addition, the ν value is smaller than for plutonium-239 so that it acts, to some extent, as inert diluent.† Consequently, the presence of plutonium-240 increases the critical mass, as may be seen by comparing the values in Table 1.2 for plutonium-239 (δ-phase) containing 4.8 and 21 wt percent, respectively, of plutonium-240.‡

DOE
b(3)

should be noted that to obtain a product with a small proportion of plutonium-240, the uranium fuel elements can be permitted to remain in the reactor for only a short time before they are removed and processed for the extraction of plutonium. As a result, the smaller the plutonium-240 content, the higher the cost of the material produced.

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†Fission of plutonium-240 exhibits an appreciable resonance for neutrons of about 1-eV energy. Otherwise, the cross sections are low except at high neutron energies, above about 0.5 MeV.

‡It is of interest that the critical mass of plutonium containing as much as 35 wt percent of plutonium-240 is still considerably less than that of uranium-235.

1.63 In evaluating the cost of the plutonium in a weapon, it must be recognized that the higher the plutonium-240 content the smaller the cost per unit mass of plutonium-239, but the larger the mass required for criticality. The actual cost per crit, which is the important criterion, is thus determined by two opposing factors. The weapon cost first decreases and then increases as the proportion of plutonium-240 is increased. However, cost is not the only factor to be taken into account in the choice of weapon material. Other considerations are the effect of plutonium-240 on boosting (Chapter 6) and on "one-point" safety (Chapter 9). Although in principle, fission weapons could be designed to use materials with a much higher content of plutonium-240 (§ 2.72), the plutonium in current use for weapons contains roughly 6 percent of plutonium-240 (see Table 1.5). This is regarded as the best compromise at the present time, although it may be subject to change with circumstances.

1.64 For a production reactor of a given type, the plutonium-240 content of the material produced depends on the neutron density (or reactor power) and the exposure time. Consequently, the quality of plutonium was at one time described in terms of megawatt-days of exposure in the reactor per ton of uranium fuel, i.e., in MWD/T units. This unit is not very precise because the power of a reactor is not uniform throughout its volume; the plutonium-240 content corresponding to a given MWD/T value will often vary with the location of the fuel in the reactor. It also depends upon the characteristics of the production reactor, so that it is different for the Hanford (graphite moderated) and Savannah River (heavy-water moderated) reactor products (see Table 1.4). An alter-

Table 1.4 CHARACTERISTICS OF PRODUCTION PLUTONIUM

MWD/T		Plutonium-240 (percent)	Neutrons/gram-sec (n/g/s)
Hanford	Savannah River		
800	1375	7.1	75
600	1000	5.6	58
400	630	3.85	40

native method for expressing quality is the number of grams of plutonium per ton of fuel removed for processing, generally indicated by g/T; the higher the g/T value the greater the plutonium-240 content. Because 1 MWD of reactor energy produces roughly

1 gram of plutonium, the values are approximately equal numerically in g/T and MWD/T units.

1.65 The two foregoing methods of indicating the quality of plutonium have now been replaced by the actual statement of the weight percent of plutonium-240. Some of the older fission weapons required plutonium of low plutonium-240 content ("clean" plutonium); the purest material generally used contained about 1.5 percent of plutonium-240.

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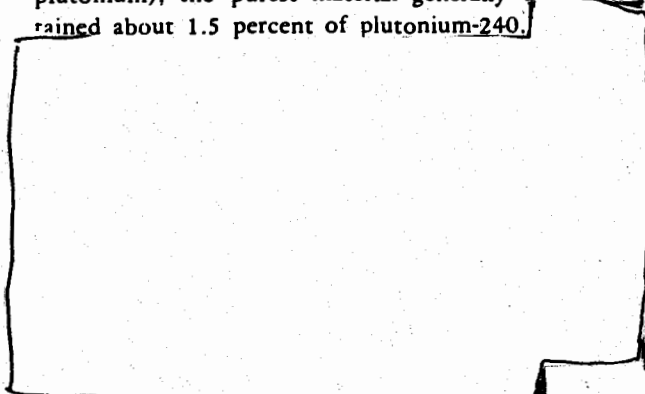
1.66 Another way of expressing the amount of plutonium-240 is based on direct measurement of the rate of neutron emission from the material. These neutrons arise mainly from the spontaneous fission of plutonium-240. The result is expressed as the number of neutrons emitted per gram per second, abbreviated to n/g/s.* The n/g/s value is seen to be approximately 10.5 times the percentage of plutonium-240. The characteristics of three production materials from the Hanford and Savannah River plants with different plutonium-240 contents are given in Table 1.4.

1.67 Continued exposure in a nuclear reactor leads to the conversion of some plutonium-240 into plutonium-241 by neutron capture, as mentioned in § 1.61. Thus, "reactor grade" plutonium, produced in power reactors, may contain 10 percent or more of the latter isotope, together with about 15 percent of plutonium-240. The fission properties, i.e., cross sections and average number of neutrons per fission, of plutonium-241 are similar to those of plutonium-239, and it would consequently be equally effective for weapons purposes. Some consideration has thus been given to the possibility of utilizing reactor grade plutonium for weapons, especially since relatively large amounts of plutonium-240 can be tolerated, if necessary.

1.68 An interesting point in this connection arises from the radioactive beta-decay of plutonium-241 (half-life 13 years) to form americium-241.

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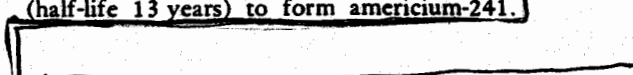
*The incorrect abbreviation "ngs" is often used in the weapons literature.

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If the plutonium were to be used in the alpha phase, however, the presence of a relatively large proportion of plutonium-241, which would decay to americium-241, could lead to a phase change to the delta form. The accompanying decrease in density would then result in dimensional instabilities. To test this point, a sample of alpha-plutonium containing 12.3 percent of plutonium-241 has been under observation since February 1964. A decrease in density from 19 to about 17.8 g/cm^3 occurred between 56 and 64 months, when the americium content was roughly 2.7 weight percent, but x-ray examination showed that the material still consisted entirely of the alpha phase. The significance of the results is uncertain and the observations are being continued.

Composition of Weapons Plutonium

1.69 In addition to plutonium-239 and -240, which are the main components, weapons-grade plutonium contains small quantities of isotopes of both higher and lower mass numbers. These are produced in a reactor by various neutron reactions either $(n,2n)$ or (n,γ). The average isotopic compositions of plutonium from Hanford and Savannah River plants reported in June 1968 are quoted in Table 1.5; these may be regarded as typical of current production.

Table 1.5   COMPOSITION OF WEAPONS-GRADE PLUTONIUM IN WEIGHT PERCENT

|	Hanford	Savannah River
Plutonium-238	<0.05	<0.05
Plutonium-239	93.17	92.99
Plutonium-240	6.28	6.13
Plutonium-241	0.54	0.86
Plutonium-242	<0.05	<0.05

Possible Weapons Materials

1.70 Several fissile nuclides, with atomic number exceeding 94, are known, but they are of no practical value for weapons purposes because of their short half-lives. A region of stability has been predicted, theoretically, however, for very heavy species in the vicinity of those having "magic numbers" of both protons and neutrons, e.g., 114 protons and 184 neutrons, i.e., $^{298}_{114}X$, and 126 protons and 184 neutrons, i.e., $^{310}_{126}X$. Such nuclides might have half-lives up to 10^6 to 10^8 years, and would be expected to be capable of sustaining a fission chain with neutrons.

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Production of Neutrons

1.73 In a fission weapon, the chain reaction is initiated by the introduction of neutrons into a critical or supercritical system. Consequently, the general methods for producing neutrons in weapons will be reviewed here. One of the simplest procedures for obtaining neutrons is by the action of alpha particles on certain light elements, notably beryllium; processes of this kind are referred to as (α,n) reactions. Recalling that the alpha particle is actually a helium nucleus, the reaction is represented by

$$^4_2He + {}^9_4Be \rightarrow {}^{12}_6C + {}^1_0n.$$
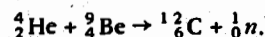
1.74 A convenient source of alpha particles, which was used extensively at one time in fission weapons, is

the radioelement polonium-210. This isotope has certain advantages, associated with corresponding drawbacks. It does not emit gamma rays, so that there is no neutron production by the (γ,n) reaction which might otherwise occur even when the polonium-210 and beryllium are separated in such a manner as to prevent access of alpha particles to the latter. Moreover, the polonium-210 is easy to produce by exposure of ordinary bismuth to neutrons in a nuclear reactor, when the (n,γ) reaction

$$^{209}_{83}\text{Bi} + ^{1}_{0}n \rightarrow ^{210}_{83}\text{Bi} + \gamma$$
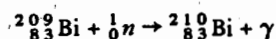
takes place. The bismuth-210 is a beta emitter with a half-life of 5 days, so that it soon decays to form polonium-210; thus,

$$^{210}_{83}\text{Bi} \rightarrow ^{0}_{-1}\beta + ^{210}_{84}\text{Po}.$$
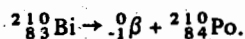
However, the production of polonium-210 in this manner means that fewer neutrons are available for the conversion of uranium-238 into plutonium-239.

1.75 As a result of the moderately short half-life of polonium-210—138.4 days—it emits alpha particles rapidly and a small quantity can thus provide a strong neutron source in conjunction with beryllium. But the short half-life is also a serious disadvantage, because the activity falls off relatively rapidly. In one year, the alpha activity, and hence the rate of neutron production, will have decreased to 18 percent and in two years to 2.6 percent of its initial value.

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1.79 Neutrons can also be produced by the action of gamma rays (or x rays) of sufficient energy on various nuclei; these processes are referred to as photoneutron or (γ,n) reactions. Since the average binding energy of a nucleon in a nucleus is about 8 MeV in many cases (Fig. 1.1), a photon of this energy will eject a neutron from most nuclei. This fact was utilized in an experimental device for initiating the fission chain in nuclear weapons (§ 5.25).

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1.80 In the great majority of weapons of recent design the neutrons required for initiation are produced by the fusion reactions described in § 1.48. The procedures involve either the D-D or D-T reactions at high temperatures (thermonuclear reactions) or the interaction of accelerated tritons with a deuterium target (electronuclear reactions). The methods used for achieving these processes in weapons are described in Chapter 5.

Production of Deuterium and Tritium

1.81 Deuterium (in the form of the mixed oxide, HDO) is present to the extent of about one atom to 6500 atoms of hydrogen, i.e., 0.015 atom percent, in ordinary water. In spite of this very small proportion, concentration of the deuterium in water is not too difficult, and heavy water of about 99.75 percent purity, i.e., 99.75 mole percent D_2O, is now produced on a large scale.

1.82 Three main processes are used to separate the two hydrogen isotopes in water; these are (a) isotopic (or chemical) exchange, (b) distillation, and (c) electrolysis. Several isotope exchange processes have been considered but the most satisfactory appears to be one involving reaction between hydrogen sulfide gas and liquid water. The exchange reaction results in a relatively higher proportion of deuterium in the liquid phase than in the gas. By utilizing a countercurrent flow of gas and liquid in several stages, in a system operating at two different temperatures, considerable enrichment in deuterium can be achieved in the liquid.

1.83 The distillation method for separating the isotopes of hydrogen depends on the fact that heavy water (D_2O) has a slightly higher boiling point, i.e., slightly lower vapor pressure, than light water (H_2O). Consequently, a partial separation can be achieved by fractional distillation, preferably under reduced pressure.

1.84 When an acid or alkaline aqueous solution is electrolyzed, the hydrogen gas liberated at the cathode contains relatively more of the lighter isotope (H_2) than does the residual water. By repeated electrolysis in stages, heavy water of a high degree of purity can be obtained.

1.85 Each of the three foregoing procedures has certain advantages under appropriate conditions. Hence, in the production of heavy water in quantity, ordinary water is first partially enriched in deuterium by the isotopic exchange process with hydrogen sulfide. The deuterium is further concentrated by fractional distillation of the enriched water under reduced pressure, and then it is brought up to 99.75 percent purity by electrolysis. Deuterium gas can be released from heavy water by any of the familiar chemical processes used to prepare hydrogen gas. It can then be very simply converted into any compound that may be required, e.g., uranium deuteride (UD_3), lithium deuteride (LiD), etc., for weapons applications.

1.86 The third isotope of hydrogen, i.e., tritium, is a radioactive beta emitter, with a half-life of 12.33 years. It is found in natural waters to an insignificant extent only and the cost of extraction would be prohibitive. Consequently, tritium is produced by nuclear reactions resulting from the exposure of lithium (as a suitable compound) to neutrons in a reactor. The less abundant lithium-6 isotope, present to the extent of 7 atomic percent in natural lithium, readily captures slow neutrons and undergoes the (n,α) reaction

$$^6_3\text{Li} + ^1_0 n \rightarrow ^4_2\text{He} + ^3_1\text{H (or T)}$$
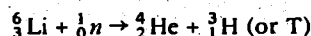
with the formation of tritium. The more common isotope, lithium-7, reacts with fast neutrons to some extent and this process also leads to the production of tritium; thus,

$$^7_3\text{Li} + ^1_0 n \rightarrow ^4_2\text{He} + ^3_1\text{H (or T)} + ^1_0 n.$$
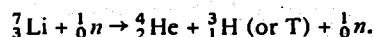
1.87 After irradiation by neutrons for a length of time, the lithium compound is removed from the reactor and the gases, consisting mainly of tritium and helium, with other hydrogen isotopes as impurities, are separated from the residual solid. Purification is achieved by a gaseous diffusion process. Compounds of tritium can be prepared from the gas by reactions similar to those employed for deuterium and ordinary (light) hydrogen.

Enrichment of Lithium

1.88 For the production of tritium, described above, it is not necessary to separate the lithium-6 from the more abundant lithium-7. Natural lithium is consequently used for this purpose. For certain applications, e.g., in some thermonuclear weapons, however, it is required that the lithium, which is employed as a hydride,* be enriched in the lighter isotope. Such material, with a lithium-6 enrichment of up to 90 percent, is produced by the counter current flow of

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*Used generically, the term hydride refers to a binary compound containing any of the isotopes of hydrogen.

lithium amalgam, Li(Hg), and an aqueous solution of lithium hydroxide, Li^+OH^-, in a cascade of ☐ packed columns. The amalgam is obtained by the electrolysis of lithium hydroxide solution using a mercury cathode.

1.89 The lighter isotope is concentrated in the amalgam by the exchange reaction

$$^7Li(Hg) + {}^6Li^+ \rightleftharpoons {}^7Li^+ + {}^6Li(Hg),$$
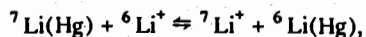
and is recovered by decomposing the amalgam with pure water in the presence of a graphite catalyst, i.e.,

$$^6Li(Hg) + H_2O = {}^6LiOH + Hg + \tfrac{1}{2}H_2.$$
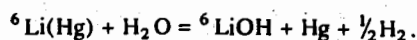
A portion of the resulting (enriched) lithium hydroxide is drawn off as the product and the remainder is refluxed to the cascade. The recovered mercury is returned to the amalgam maker at the head of the cascade.

1.90 The hydroxide enriched in lithium-6 is neutralized with hydrochloric acid; the resulting aqueous solution of lithium chloride is concentrated by evaporation and the salt is allowed to crystallize. Lithium metal is then obtained by electrolysis of the molten lithium chloride. For the production of various lithium hydrides enriched in ^6Li, the metal is reacted at a suitable temperature with the appropriate isotope of hydrogen under pressure.

Chapter 2

THE FISSION PROCESS IN WEAPONS

INCREASE OF NEUTRON POPULATION

The Multiplication Rate: Alpha

2.01 No matter how it originates, an explosion is associated with the very rapid liberation of a large amount of energy within a restricted space. If the energy is to be produced by fission, then an essential condition for explosion is a very high neutron density, since the rate of fission, and hence the rate of energy release, is proportional to the number of free neutrons per unit volume. It is of interest, therefore, to examine the factors which lead to a high neutron density, since these will form a basis for fission weapon design.

2.02 In accordance with the definition of the effective multiplication factor, k, given in § 1.19, it follows that for every n neutrons present at the beginning of a generation there will be nk neutrons at the beginning of the next generation, so that the gain of neutrons is $n(k-1)$ per generation. The rate of gain, dn/dt, may then be obtained upon dividing the actual gain by the average time, τ, between successive fission generations; hence,

$$\frac{dn}{dt} = \frac{n(k-1)}{\tau}.$$ (2.1)

Equation (2.1) will be strictly correct only if delayed neutrons play no part in maintaining the fission chain. As already stated (§ 1.13), this condition is applicable, to a good approximation, to nuclear fission weapons.

2.03 The quantity $k-1$, which is the excess number of available neutrons per fission, may be represented by x, i.e.,

$$x \equiv k - 1,$$ (2.2)

and then equation (2.1) becomes

$$\frac{dn}{dt} = \frac{x}{\tau} n.$$ (2.3)

The time rate of increase (or decrease) in neutron population can be expressed in the general form

$$\frac{dn}{dt} = \alpha n,$$ (2.4)

where α is the specific rate constant for the process which is responsible for the change in the number of neutrons. In nuclear weapons work this constant is called the multiplication rate or merely "alpha." Comparison of equations (2.3) and (2.4) shows that for a fission chain reaction

$$\alpha = \frac{x}{\tau}.$$ (2.5)

Since τ is the time per fission generation and x is a number (per fission), the units of α are generations per unit time or generations/time.

2.04 The foregoing results are applicable regardless of whether x, and hence α, is positive, zero, or negative. For a subcritical system, $k < 1$ (§ 1.20), i.e., $k - 1$ is negative; in these circumstances x is negative and so also is α. It follows from equation (2.4) that dn/dt is then negative and the number of neutrons in the system will decrease with time. Consequently, in agreement with previous conclusions, the fission chain in a subcritical system will eventually die out because of the steady decrease in the neutron population. When the system is just critical, $k = 1$, and x and α are both zero; the number of neutrons will thus remain constant. Finally, for a supercritical system, $k > 1$, and x and α are positive; there will then be a steady increase in the neutron population. Since dn/dt is proportional to n, by equation (2.4), it is evident that in a supercritical system, the number of neutrons will grow at increasingly faster rates as n increases.

2.05 Another aspect of the significance of α becomes apparent when equation (2.4) is written in the form

$$\frac{dn}{n} = \alpha \, dt.$$

If α is assumed to remain constant, an acceptable assumption for a simple fission device somewhat before explosion, this expression can be readily integrated between the time limits of zero, when the number of neutrons present is n_0, and t, when the number is n. The result is

$$n = n_0 e^{\alpha t}, \qquad (2.6)$$

where, as usual, e is the base of natural logarithms. This expression, like those given above, is applicable regardless of whether α is positive, zero, or negative. If α is known, equation (2.6) can be used to calculate the neutron population at any time t relative to the value at any arbitrary zero time. It can also be seen from equation (2.6) that $1/\alpha$ is the time period during which the number of neutrons changes by a factor e; consequently, $1/\alpha$ is often referred to as the e-folding time, i.e., the time in which there is an e-fold change in the neutron population.*

Measurement of Alpha

2.06 The value of α is a highly important quantity in weapons design, as will shortly be apparent. It is generally obtained from a computer calculation based on the neutronic and hydrodynamic characteristics of the system, but there are uncertainties involved and experimental measurements are desirable. In weapons tests, the alpha measurement is one of the most important diagnostic requirements. Calculations are then normalized to agree with the observed values for each type of implosion system.

2.07 The determination of α involves the measurement of the time variation of the neutron population in the exploding system during a period of a microsecond or less. It follows from equation (2.6) that if log n is plotted against time, the slope of the curve will give α. The quantity measured is not n, the actual neutron population, but some parameter which is proportional to it. Because the number of neutrons emitted per fission varies, the relationship between the total number of neutrons and the total fissions is statistical rather than exact. More important is the fact

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*It will be shown in §2.16 that x is about 1, so that $\alpha \approx 1/\tau$ by equation (2.5); hence, $1/\alpha$ is approximately equal to the generation time. The neutron population thus changes by about a factor of e (= 2.72) in each generation.

that, since some neutrons are lost by nonfission captures and by escape, not all the fission neutrons are available for causing fissions in the next generation. However, at any given instant, the fission rate is proportional to n at that instant, and so the time variation of any quantity proportional to the fission rate can be used to determine α. A convenient quantity of this kind which lends itself to ready measurement is the gamma-ray flux emitted from the exploding weapon.

2.08 One method for determining α is to place a number of gamma-ray scintillation detectors fairly close to the device to be exploded. These detectors are connected by coaxial cables to oscilloscopes located at some distance. When the explosion occurs the detectors will respond to the gamma-ray flux in their vicinity and will send appropriate signals to the oscilloscopes before being destroyed by the explosion. Instead of the gamma-ray flux, the neutrons escaping from the system have sometimes been used to determine α. The rate of escape may be regarded as being proportional to the neutron population in the weapon undergoing fission.

2.09 During the design phase of a weapon, α is computed theoretically and laboratory experiments, which do not involve nuclear explosions, are used to provide normalization points for the calculations. Since a supercritical (or even a critical) mass cannot be handled safely under ordinary conditions, experimental measurements of α are made with a mass that is slightly subcritical. In the Rossi method, a single neutron is introduced at intervals into a slightly subcritical system. One (effective) neutron starts a fission chain which soon dies out; then another neutron starts a second chain which dies out, and so on for the duration of the experiment. The counters are thus exposed over an appreciable period to neutrons from a large number of chains one at a time, and so the method is statistical in character. The neutron counting is carried out by means of a multichannel time-delay circuit. The first channel is activated by entry of a neutron; then, after a pre-determined interval, a second channel is activated for a short time; after a further interval, a third channel is activated, and so on for intervals ranging from 0.25 μsec to a total of a few milliseconds. From the accumulated counts for the various channels, the time rate of decay of the prompt neutrons after fission can be determined and α calculated from equation (2.6).

2.10 The values of α obtained as described above apply to the particular subcritical system used. In order to estimate what α would be in a supercritical

(weapon) assembly of the same materials, an extrapolation procedure can be employed. By means of a number of measurements with different assemblies, α can be determined as a function of the number of (prompt) crits present. The values could then be extrapolated to give α for the number of crits expected at the time of initiation of the weapon. Such a procedure is highly uncertain since it involves extrapolation from a series of assemblies which are slightly subcritical to one which is highly supercritical. As previously noted, detailed machine calculations, normalized to experimental results, are currently used to estimate α values.

Conditions for Nuclear Explosion

2.11 It was pointed out in § 2.01 that the essential condition for a nuclear fission explosion is a very high density of free neutrons. Equation (2.6) shows that if a large density n is to be attained within a short time, α must be large. In general, the magnitude of α determines the efficiency of a nuclear explosion, and it is of interest to consider how α may be made as large as possible.

2.12 According to equation (1.4), k is equal to $\nu-l$, where ν is the average number of neutrons produced in each act of fission and l is the average number lost per fission by escape and in other ways. Upon utilizing this result, together with equations (2.2) and (2.5), it follows that

$$\alpha = \frac{x}{\tau} = \frac{\nu-l-1}{\tau}. \qquad (2.7)$$

In order to increase α it is consequently necessary that $\nu-l-1$ be large and τ be small. The value of ν is a specific property of the fissile material for fission by neutrons of a given energy. Since ν increases with neutron energy, it would appear to be advantageous from this standpoint if most of the fissions in a weapon were caused by fast neutrons. The relative neutron loss, l, can be decreased, e.g., by assembly or compression or by using a neutron tamper. If the volume increases, the loss of neutrons per fission increases and α will decrease accordingly.

2.13 A highly significant contribution to the magnitude of α is made by the fission generation time, τ. This is approximately equal to λ, the fission mean free path of the neutrons in the core material, divided by the (average) speed, v, of the neutrons causing fission, i.e.,

$$\tau \approx \frac{\lambda}{v}, \qquad (2.8)$$

so that in a fission weapon it is desirable that the ratio λ/v be as small as possible. It should be noted that the fission mean free path in equation (2.8) is not the same as the total mean free path for all interactions used in § 1.43. The λ employed here is the average distance a neutron travels before it is captured in a fission reaction.

2.14 The fission mean free path is equal to $1/N\sigma_f$, where N is the number of fissile nuclei per cm^3 and σ_f is the fission cross section.* Hence, from equation (2.8),

$$\tau \approx \frac{1}{N\sigma_f v}, \qquad (2.9)$$

and the generation time is inversely proportional to the product $\sigma_f v$. The value of σ_f decreases as v increases but the product is $\sim 2 \times 10^9$ (in 10^{-24} cm^3/sec units) for fast neutrons of 1-MeV energy compared with 10^8 for slow neutrons. Hence, the fission generation time is appreciably shorter for fast than for slow neutrons.

2.15 From the information already given, it is possible to make rough, order-of-magnitude estimates of τ and α. For uncompressed uranium-235 or plutonium-239, the number, N, of nuclei per cm^3 is approximately 0.5×10^{23} and, as stated above, $\sigma_f v$ for fast-neutron fission is $2 \times 10^9 \times 10^{-24} = 2 \times 10^{-15}$ cm^3/sec. Hence, from equation (2.9),

$$\tau \approx 1/(0.5 \times 10^{23})(2 \times 10^{-15}) \approx 10^{-8} \text{ sec.}$$

The generation time for fast neutrons in a fission weapon is thus roughly 10^{-8} sec, i.e., 1 shake, or 0.01 microsecond, i.e., 0.01 μsec.

2.16 As seen from Table 1.1, ν is about 2.5 to 3, and the loss, l, of neutrons per fission may be taken to be 0.5 to 1. Hence, by equation (2.2), the value of x is close to unity for a highly supercritical system. It follows, therefore, from equation (2.7) that

$$\alpha \approx \frac{1}{\tau} \approx 10^8 \text{ gen/sec (or } 10^8 \text{ gen sec}^{-1}).$$

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*For the present purpose it is sufficient to regard the cross section as the effective area in sq cm of a nucleus for a particular reaction (or reactions). Cross sections vary with the neutron energy and the fission cross sections for uranium-235 and plutonium-239 have been measured over a large energy range.

Thus, for fast-neutron fission α is approximately 10^8 gen/sec.

2.17 Values of α are commonly expressed either in generations per shake, i.e., gen/shake, or per microsecond, i.e., gen/μsec. The rough calculations made above indicate that, in a weapon, α is of the order of 1 generation per shake or 100 generations per μsec. Experimental measurements, both in the laboratory and at weapons tests, show that this is indeed the case.

2.18 It is evident that in order to achieve an efficient nuclear explosion, fission should be brought about by fast neutrons, as far as possible. For such neutrons, the factors ν and τ, and to some extent l, favor a high value of α and, hence, a rapid increase in the neutron population. It would appear desirable, therefore, to keep elements of low mass number, which slow down neutrons more effectively than heavier ones, out of the core, in particular, as well as the reflector of a fission weapon. At one time, special precautions were taken to do so, but in recent years the element beryllium, mass number 9, has been widely used as a reflector material in small weapons because of its satisfactory neutronic properties (see Table 1.2) and low mass.

2.19 According to equation (2.9), the fission generation time for neutrons of a given energy (or velocity) is inversely proportional to the number, N, of fissile nuclei per cm^3. It follows, therefore, that τ is inversely proportional to η, the core compression ratio; thus

$$\tau \propto \frac{1}{\eta}. \tag{2.10}$$

Consequently, the generation time can be decreased, and the value of α increased proportionately, by compression of the core material.

2.20 In addition to the effect on τ, compression also causes a marked decrease in l for the reason given in § 1.40. This also contributes to an increase in α, as follows from equation (2.7). It is seen, therefore, that compression of the core will cause an increase in α because of the decrease in both τ and l.

Explosion Time

2.21 Suppose that a slightly subcritical mass of fissile material is compressed, as in an implosion device. The system will pass through a critical configuration, referred to as "first critical," and then become increasingly supercritical. After reaching a stage of optimum (or maximum) supercriticality, corresponding to optimum assembly or maximum compression, the system would subsequently become less and less supercritical. If a fission chain is initiated by the introduction of neutrons, the variation of α with time would be qualitatively as indicated in Fig. 2.1.
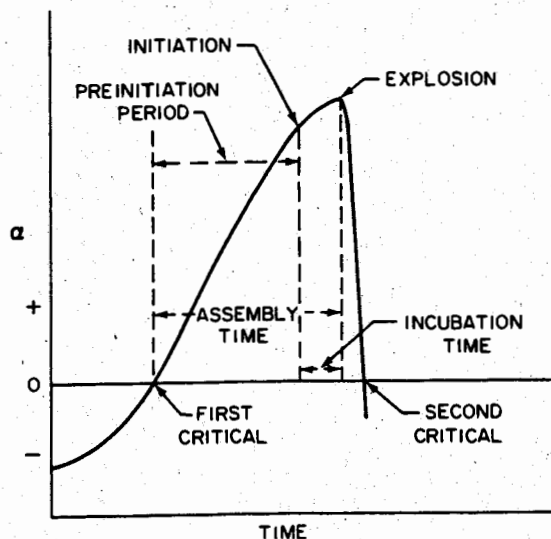

Figure 2.1

2.22 If a divergent chain reaction, leading to an explosion, is to be established in the system, at least one neutron must be present while the system is supercritical, i.e., after first critical. For maximum efficiency, the explosion should occur as close as possible to maximum supercriticality and the neutrons should be introduced at an appropriate earlier time. For the present general discussion, it will be assumed that the fission chain is initiated before optimum supercriticality, as is the usual situation for implosion systems.

2.23 Once the fission chain has been started, energy release will commence. The motion of the fissile material in the assembly process will not be affected significantly, however, until the energy generated by fission produces appreciable mechanical forces. After a certain time, the increase of temperature accompanying the release of the fission energy will generate expansion pressures high enough to stop the motion of the assembly, and subsequently disassembly will begin. The point at which assembly ceases and disassembly commences is known as the "explosion time." The interval between initiation of the fission chain by a neutron and explosion time is referred to as the

"incubation time" (Fig. 2.1). The time between first critical and explosion (or maximum supercriticality) is here called the "assembly time."*

2.24 Because of the rapid increase in the neutron population during the propagation of the fission chain reaction, the fission energy is released at a very high rate. The heat generated causes the temperature to increase and the fissile material expands (or disassembles) rapidly. Soon the volume becomes so large that the system becomes subcritical and the divergent chain reaction ceases. The point at which the material passes through the critical stage during expansion is called "second critical" (§ 2.59).

2.25 The number of fission generations corresponding to the explosion time for a spherical (imploded) core can be calculated in a semiquantitative way in the following manner.† If the reasonable assumption is made that the pressure in the assembly has a parabolic distribution, then the pressure P at a distance r from the center of the core of radius R is

$$P = P_0\left(1 - \frac{r^2}{R^2}\right). \qquad (2.11)$$

where P_0 is the pressure at the center. If $\bar{P}$ is the average pressure in the core then, as shown in the appendix to this chapter,

$$\bar{P} = \frac{1}{5}\rho R a, \qquad (2.12)$$

where ρ is the density of the core material and a is the acceleration at its surface.

2.26 If the change in velocity at the surface of the core at a time t after initiation of the chain reaction is represented by Δv, then

$$\Delta v = \int_0^t a\,dt. \qquad (2.13)$$

Furthermore, with an exponentially increasing neutron population, as indicated by equation (2.6), the ac-

celeration, as well as the energy density and the pressure, will vary as $e^{\alpha t}$; hence,

$$a = Ae^{\alpha t},$$

where A is a constant. If this result is substituted into equation (2.13), it is seen that

$$\Delta v = \frac{a}{\alpha}. \qquad (2.14)$$

2.27 Suppose the unperturbed velocity of the core surface during assembly, i.e., before it is affected by the fission energy, is v_0; then the assembly motion will be halted when

$$\Delta v = v_0, \qquad (2.15)$$

and this corresponds to explosion time. Consequently, from equations (2.14) and (2.15), at explosion time

$$a = v_0\alpha,$$

and from equation (2.12)

$$\bar{P} = \frac{1}{5}\rho R v_0 \alpha. \qquad (2.16)$$

This is the average pressure in the core at explosion time.

2.28 The application of equation (2.16) may be illustrated by considering the hypothetical case of a core with a density, ρ, of 20 g/cm³, and a radius, R, of 5 cm. The unperturbed rate of assembly, v_0, may be

DoE b(3)

and α is set at 1 gen/shake, i.e., 10^{8} gen/sec. If these values are substituted into equation (2.16), it follows that at explosion time

DoE b(3)

2.29 At explosion time, the core will be effectively a gas at very high pressure and it may be considered to

---

*In weapons test, a measured time interval is that between firing the HE system and the first appearance of gamma rays from the explosion; it is called the "HE transit time" or simply the "transit time."

†The purpose of this treatment is merely to provide a general understanding of the core behavior. In weapons design studies, more exact calculations are made with the aid of computers.

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‡The unit 1 bar is equivalent to a pressure of 10^6 dynes/cm²; the megabar, i.e., 10^6 bars, is then 10^{12} dynes/cm². The standard atmospheric pressure (760 mm of mercury) is 1.013 bars; thus a pressure of 1 atm is approximately 1 bar.

have been compressed adiabatically. The energy, E, of the core will then be given by

$$E = \frac{PV}{\gamma - 1}. \qquad (2.17)$$

For an ideal gas, γ is $5/3$, but for the core it is probably nearer to $3/2$. Hence, using the value of $\bar{P}$ derived above, it is found that at explosion time the energy per unit volume is

DOE b(3)

$$\bar{E}_v =$$

provided the loss of energy from the system prior to this time is not significant.

2.30 Since the density of the core has been taken to be 20 g/cm³, the average energy per gram, $\bar{E}_g$, at explosion time in the hypothetical case under consideration is

DOE b(3)

$$\bar{E}_g =$$

DOE b(3)

2.31 The radius of the core is 5 cm and hence the volume, V, is $\frac{4}{3}\pi (5)^3 = 524$ cm³. The total energy, E, released by explosion time, is, therefore,

DOE b(3).

$$E = \bar{E}_v V =$$

which is equivalent (§ 1.15) to

DOE b(3)

                                    ons TNT.

Consequently, an energy   ons of
TNT is released by explosion time in the case under consideration.

DOE b(3)

2.32 The amount of fission energy contributing to the explosion is equivalent to 7×10^{-24} kiloton (or 7×10^{-21} ton) per fission (§ 1.15). Hence, at explosion time the number of fissions that have occurred is

DOE b(3)

Number of fissions =

=

2.33 It was seen in § 2.16 that x is approximately equal to unity in a weapon and then α is roughly equivalent to $1/\tau$. Consequently, equation (2.6) may be written as

$$n \approx n_0 e^{t/\tau} = n_0 e^g,$$

where t/τ, represented by g, is the number of generations in which the neutron population increases from n_0 to n. If it is postulated that the fission chain is initiated by a single neutron, the number of neutrons present after g generations is

$$n \approx e^g.$$

As an approximation, it may be assumed that the number of fissions after g generations is equal to the neutron population, n, at that time. Hence, it follows that in the hypothetical example used above explosion time occurs   DOE b(3)

2.34 It will be evident from the foregoing treatment that the explosion time depends on such quantities as ρ, R, v_0, and α, and hence it will vary with the design of the fission weapon

DOE b(3)

2.35 In this derivation of explosion time it has been assumed that the fission chain is initiated by one neutron. The results would not be affected significantly if a few neutrons were present at initiation time. In some types of initiators, however, many neutrons are supplied to start fission chains. The number of generations at explosion time will then be less than obtained above

DOE b(3)

2.36 Another point that should be mentioned about these calculations is the assumption that the

total number of fissions which have occurred by a particular time is equal to the neutron population at that time. This assumption represents a simplification of the actual situation. A more precise treatment, however, leads to results in agreement with those obtained above.

Rate of Energy Release

2.37 Since the explosion energy released per fission is equivalent to 7×10^{-21} ton of TNT and the total number of fissions at g generations may be set equal to e^g, assuming initiation by a single neutron, it follows that

Energy released by g generations $\approx 7 \times 10^{-21}\ e^g$

$$= 7 \times 10^{-21} \times 10^{g/2.3} \text{ tons TNT.}$$

The energy releases after various numbers of generations have been calculated from this expression; some of these are quoted in Table 2.1.

Table 2.1 TOTAL ENERGY RELEASE

No. of generations	Energy release (TNT equivalent)

DOE
b(3)

DOE
b(3)

2.38

DOE
b(3)

It is evident from the results in Table 2.1 that nearly all the energy is released in the next 7 (or so) generations. The reason is, of course, that the energy increases by a factor of $e = 2.72$ in each generation or, $e^7 \approx 10^3$ in 7 generations.

2.39 It is true, in general, that only a small proportion, usually something like 0.1 percent (1 part in 10^3) of the energy of a fission weapon is produced by explosion time; the remaining 99.9 percent is released during the next few generations. It is thus accepted that before explosion time, at [   ] generations after initiation, the energy release is too small to have any appreciable effect on the motion of the assembly. Within this period, therefore, the core volume may be assumed to remain essentially constant.

DOE
b(3)

2.40 In a simple device with a short incubation period the core volume is more or less constant

between initiation and explosion time; hence, α in a pure fission (but not boosted) device is also roughly constant. After explosion time, however, when the system expands, α decreases and the generation time increases. As a rough approximation, however, the generation time may be taken to be 1 shake. Hence, it is during a period of about 7 shakes, i.e., 0.07 μsec, that most of the energy of a fission device is produced. The release of a large amount of energy in such a very short interval of time, in a restricted volume, results in the attainment of very high temperatures—several tens of million degrees. Large pressures consequently develop in the core and rapid expansion, i.e., an explosion, occurs.

2.41 When the expansion is such that α has fallen to zero and the system passes through second critical, the self-sustaining fission chain will cease, although substantial amounts of fissile material may still be present. To attain a high efficiency, i.e., to consume as much as possible of the fissile species, expansion of the core should be delayed. This important function in pure fission weapons was filled by the inertial tamper. In boosted devices, however, the introduction of neutrons from an extraneous (D-T) source results in a considerable increase in the fission rate before the system expands significantly. The inertial tamper then appears to be less important than in an unboosted system.

EFFICIENCY OF FISSION WEAPONS

Definition of Efficiency

2.42 The efficiency, ϕ, of any weapon may be defined as the ratio of the energy actually developed when it explodes, i.e., the energy yield, to the total energy available; thus,

$$\phi = \frac{\text{Energy yield}}{\text{Energy available}}. \qquad (2.18)$$

In other words, the efficiency is the fraction of the total energy available which is actually released in the explosion. In the case of a fission weapon, this is equal to the ratio of the quantity of fissile material which actually suffers fission to the total amount present in the weapon. The efficiency of a weapon is generally expressed as a percentage, and so it is equal to ϕ, as defined by equation (2.18), multiplied by 100. There are several factors which determine the efficiency and some of these will be discussed below.

2.43 It is of interest to mention, in passing, that the efficiency of the earliest implosion weapons was

DOE
b(3)

the gun-type systems (§ 1.36). This accounts for the fact that devices of the latter type have occupied a secondary place in the development of fission weapons, except for special purposes.

Calculation of Efficiency: Computer Methods

2.44 At the present time, the efficiency or, more correctly, the energy yield of a weapon is determined by computer calculations based on codes which have been developed to represent the behavior of the fission chain system. The calculations usually start at or immediately before explosion time, when significant mechanical effects may be assumed to begin. From initiation up to this time, the material is essentially stationary or is still being compressed and only a small proportion of the total energy has been liberated. The treatment takes into account the neutronic behavior, the hydrodynamics, and heat flow; the motion of a series of concentric shells (or "mass points") is followed until the rate of energy release by fission has fallen almost to zero. The total yield includes the energy released after the system has expanded and become subcritical. Although self-sustaining chain propagation is no longer possible, convergent-chain interaction of the many neutrons and fissile nuclei still present will result in considerable energy production. This may amount to some 30 percent or more of the total yield.

The Bethe—Feynman Formula

2.45 Prior to the development of computing machine procedures, and before data were available from test explosions for comparison and normalization purposes, fission weapon efficiencies were estimated by the method of Bethe and Feynman. The basic formula is admittedly approximate, since it involves several simplifying assumptions. However, its derivation is useful in the respect that it provides a model of the explosion of a fission weapon and indicates, qualitatively at least, some of the factors which affect the efficiency of the explosion. The treatment given below is applicable to pure fission systems and not after boosting occurs.

2.46 As a result of the energy liberated in fission, very large pressures (10–1000 megabars) are developed in the core, and the core-tamper interface consequently receives a large outward acceleration. This causes highly compressed tamper material to pile up just

ahead of the expanding interface, in an effect referred to as the "snowplow" phenomenon, because of the similarity to the piling up of snow in front of a snowplow. The inertia of the compressed tamper delays expansion of the core, so that a considerable pressure gradient builds up from the center of the core to its outer surface.

2.47 Furthermore, because of the delayed expansion, it may be supposed that the volume of the compressed (supercritical) core remains essentially constant during the [   ] or so generations following initiation of the fission chain, i.e., up to explosion time. After this interval, almost the whole of the energy is released within an extremely short period, during which time the supercritical core expands rapidly until it becomes subcritical. Although there is an appreciable release of energy even while the system is subcritical, as mentioned in § 2.44, it will be postulated that energy production ceases when the dimensions are just subcritical. It will be assumed, further, that no energy escapes during the short period of expansion from maximum supercriticality to the point where the system becomes subcritical.

DOE
b(3)

2.48 Let R be the radius of a spherical core at the point of maximum supercriticality; then, in accordance with the postulate made above, this will remain unchanged until explosion time. Subsequently, the energy density of the system becomes so large that mechanical effects begin and the core starts to expand. Suppose that when the core has expanded by a fraction δ, so that the radius is $R(1 + \delta)$, the system is just critical (Fig. 2.2); beyond this point it will be subcritical. The self-sustaining fission chain will then end and, in accordance with the approximation postulated above, there will be no further release of energy.
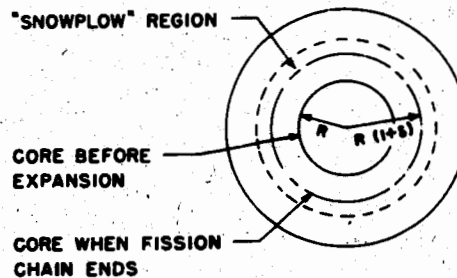

Figure 2.2

2.49 Consider a thin shell of material in the core, of volume dV and thickness dR; the cross sectional area of the shell is then dV/dR. If dP is the pressure difference on the two sides of this shell, caused by the liberated fission energy, the net outward force, dF, to

which the shell is subjected, i.e., pressure × area, is then

$$dF = dP \frac{dV}{dR} = \frac{dP}{dR} dV, \qquad (2.19)$$

where dP/dR is the pressure gradient in the given shell. As a reasonable approximation, it may be supposed that the pressure gradient is essentially constant across the core radius, so that

$$\frac{dP}{dR} \approx \frac{P}{R}, \qquad (2.20)$$

where P is the total difference in pressure from the center of the core to the outer surface before expansion occurs. Hence, from equations (2.19) and (2.20),

$$dF \approx \frac{P}{R} dV. \qquad (2.21)$$

2.50 The time required for the core to expand from radius R to $R(1 + \delta)$, i.e., a distance of $R\delta$, is about 7 generations, as seen in §2.38. However, as a rough approximation, this may be taken as $1/\alpha$, where α is the multiplication rate just prior to explosion time. The mean outward acceleration of the core material, and of the shell dV, may consequently be expressed as $R\delta\alpha^2$. The mass of the shell is ρdV, where ρ is the core density; hence, by Newton's second law of motion, i.e., force = mass × acceleration, the force dF acting on the shell is given by

$$dF \approx \rho dV \times R\delta\alpha^2.$$

Upon comparing this result with equation (2.21), it is seen that

$$P \approx \rho R^2 \alpha^2 \delta. \qquad (2.22)$$

2.51 As in the calculation of the explosion time, the total energy of the core, assuming there is negligible loss during the initial expansion, is expressed by equation (2.17), namely,

$$E = \frac{PV}{\gamma - 1}, \qquad (2.23)$$

where γ is the ratio of the specific heats of the gas. Using equation (2.22) for P and writing M/ρ for the volume of the core, M being the mass, equation (2.23) becomes

$$E \approx \frac{MR^2\alpha^2\delta}{\gamma - 1}. \qquad (2.24)$$

2.52 If ϵ is the energy released in the complete fission of unit mass of core material, then the total energy available in the core is $M\epsilon$, and the efficiency, according to equation (2.18), is $E/M\epsilon$, where E is given by equation (2.24); consequently,

$$\phi = \frac{E}{M\epsilon} \approx \frac{R^2\alpha^2\delta}{(\gamma - 1)\epsilon}. \qquad (2.25)$$

It should be pointed out that in the foregoing derivation no allowance has been made for depletion of the core material as fission proceeds. For low efficiencies, to which most of the other approximations made are applicable, the depletion is not significant and can be neglected. Moreover, no allowance has been made for the inertial effect of the tamper on the efficiency. For the present purpose, which is to obtain a qualitative guide to some of the factors determining the efficiency, this can also be ignored. Hence, replacing the quantity $1/(\gamma - 1)\epsilon$ by a constant, K, equation (2.25) can be written as

$$\phi \approx KR^2\alpha^2\delta, \qquad (2.26)$$

which is a version of the Bethe–Feynman formula that was developed for slightly supercritical systems. The efficiency of a fission weapon is seen to depend on the factors R, α, and δ.

2.53 Since the efficiency of a fission weapon may be expected to increase as R^2 (at constant density), it would be advantageous for the core to be large at the time of the initiation of the fission chain. One way in which this can be achieved in practice, e.g., in a gun-type weapon, is to bring together subcritical masses which are designed to contain a large total mass of fissile material. Thus, for a given compression or, especially, for no compression, the efficiency would be expected to be greater the larger the mass of the assembled core.

2.54 In general, the most important factor in determining the efficiency of a fission weapon is α, and the latter increases in proportion to the compression (§2.19). The efficiency, according to equation (2.26), will thus be related, approximately at least, to η^2 (or to $\eta^{1.7}$ if the effect of tamper compression is included (§1.47)). In addition, although of lesser significance, the effect of compression on δ must be taken into account; the more highly compressed the core material

at the time of initiation (or at explosion time), the farther will be the distance the core surface must travel during the expansion phase before the supercritical system becomes subcritical. Increased compression should thus result in a marked gain in the efficiency of a fission weapon. It is this fact which is largely responsible for the much higher efficiencies of implosion systems than of gun-type devices.

DoE
6(3)

2.55 The effect of increasing compression in a simple (unboosted) implosion system is indicated by the data in Table 2.2 which are based partly on experimental observations and partly on calculation.

Table 2.2 EFFECT OF COMPRESSION ON EFFICIENCY

DoE
6(3)

Average compression		Efficiency (percent)
Core	Tamper	

DoE
6(3)

The attainment of high compression was an important objective in pure fission weapon design. In the earliest (solid-core) devices the improvement in efficiency was the main purpose. In more recent (hollow-core boosted) weapons, however, the principal objective is to make possible the design of compact systems in which both the high explosive (§ 1.42) and the fissile material have low masses. The number of crits at maximum compression is not large, so that α is relatively small before boosting. The initial efficiency of the fission chain is, therefore, also small but the total yield is greatly increased by the boosting.

INITIATION TIME AND PREINITIATION

Unboosted Implosion Weapons

2.56 It was indicated in Chapter 1 that pre-initiation of a fission weapon, i.e., initiation of a chain reaction before the optimum time, could lead to a decrease in efficiency. The reason for this will be clear from the results derived in the preceding section. According to equation (2.26), the efficiency is proportional to α^2 and hence initiation at a time that will lead

to explosion before α has attained its optimum value (see Fig. 2.3), i.e., preinitiation, would inevitably result in a loss of efficiency in a simple (unboosted) fission device.

2.57

DoE
6(3)

Fortuitously, major advances in the design of weapons with improved characteristics led at the same time to a decrease in preinitiation probability. Such advances included the use of composite cores containing both plutonium and oralloy, with smaller amounts of plutonium than in all-plutonium cores, and the development of small weapons with hollow cores.

2.58 The problems of preinitiation and its avoidance are conveniently considered in relation to the assembly and incubation times defined in § 2.23. In an unboosted, solid-core device, the system is initially subcritical and upon assembly it passes promptly through first critical and then becomes supercritical, as already described. The value of α is negative before first critical, it is zero at first critical, and becomes increasingly positive during assembly (Fig. 2.3). Prior to initiation, of course, there is no neutron multiplication and hence no actual α, but the curve in Fig. 2.3 shows the potential α at various times during assembly.
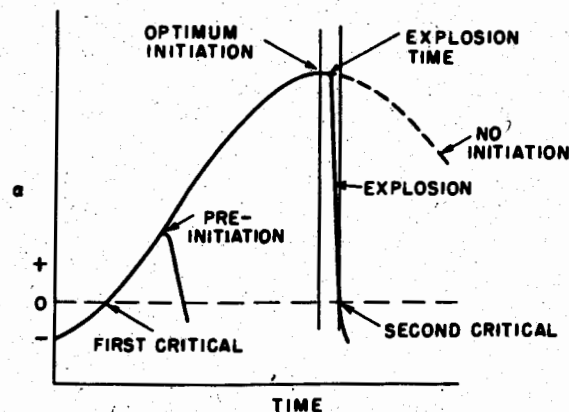

Figure 2.3

2.59 Just before optimum assembly, where α is approaching its maximum value, neutrons are injected into the highly supercritical system. The divergent

fission chain is initiated and energy is released. Between initiation time and explosion time the volume is more or less constant and so also is α (§ 2.40). Rapid expansion, however, results in loss of neutrons at an increasing rate and causes the core to become less and less supercritical; hence α decreases rapidly after explosion time. When second critical is reached, both the neutron population and the rate of fission have their maximum values. Beyond second critical a self-sustaining chain reaction is no longer possible; nevertheless, considerable amounts of energy are produced by convergent chains in the subcritical system.

2.60 It is seen from Fig. 2.3 that for the particular type of simple fission weapon under consideration, the assembly time, i.e., between first critical and explosion, is long compared with the incubation time i.e., between initiation and explosion time.

DOE b(3)

Because of the longer assembly times, preinitiation is more probable in gun-type than in implosion systems. Some aspects of preinitiation in weapons of the former type are discussed later (§ 2.73).

2.61

DOE b(3)

DOE b(3)

DOE b(3)

DOE
b(3)

2.71

DOE
(3)

Use of Dirty Plutonium in Implosion Weapons

2.72

DOE
2(3)

Gun-Type Weapons

2.73 In a gun-type weapon, the lack of compression makes it desirable for initiation to take place at or close to the time when assembly

DOE
d(3)

DOE
b(3)

Consequently, although plutonium-239 with an appreciable proportion of plutonium-240 could be used in implosion systems, even of the solid-core, unboosted type, it is unsuitable for use in conventional gun-assembly devices, as the calculations given below will show.

2.74 Apart from the possible presence of a "flood" of neutrons, e.g., as the result of the explosion of another nuclear weapon in the vicinity of a given weapon, the chief sources of background neutrons, which could cause preinitiation in a gun-assembly weapon, are spontaneous fission and (α,n) reactions with light elements. In uranium-235 (oralloy) the rate of spontaneous fission is relatively small, as will be seen shortly, but (α,n) reactions with light-element impurities could produce an appreciable neutron background.

DOE
b(3)

2.75 Let P_1 be the probability that a background neutron will be available in the fissile material during the period that it is supercritical, i.e., in the preinitiation period, and let P_2 be the probability that this neutron will be able to start a fission chain. The preinitiation probability, P, is then given by

$$P = 1 - e^{P_1 P_2}$$

$$\approx P_1 P_2, \qquad (2.27)$$

the approximate form being applicable when $P_1 P_2$ is small, as it is in cases of interest. Actually P_2 is a function of time and both P_1 and P_2 depend, to some extent, on the position of the neutron and on other factors. For the present purpose, however, which is to draw general conclusions only, specific values will be assigned to P_1 and P_2. A background neutron entering a fissile assembly may escape altogether without being absorbed, or it may be captured in a nonfission

DOE
b(3)

reaction, or it may initiate a fission chain. Although the probabilities of these three processes are by no means equal, it is sufficient to postulate here that P_2 has a constant average value of 0.3 over the preinitiation period. Hence, for the purpose of making rough estimates, equation (2.27) may be written as

$$P \approx 0.3\, P_1. \qquad (2.28)$$

2.76 As a result of spontaneous fission, uranium-235 emits, on the average, about 0.70 neutron per kilogram per sec, whereas uranium-238 produces roughly 15 neutrons/kg-sec. Consequently, the neutron background in ordinary oralloy (93.2 weight percent uranium-235) is approximately 1.6 neutrons/kg-sec.

DOE b(3)

2.77 According to the numbers quoted above, more than half of the background neutrons in ordinary (93.2 percent) oralloy arise from the uranium-238 present.

DOE b(3)

2.78 Because of the high preinitiation probability, it is necessary to keep other background neutrons to a minimum in gun-assembly devices. It is for this reason

DOE b(3)

It is seen, therefore, that a conventional gun-type weapon, based on plutonium with assembly brought about by a propellant explosive, is completely out of the question. It was the realization of this fact, when plutonium became available, that led to the development of implosion systems (§ 1.39), together with the expectation that the compression achieved by implosion would lead to greater efficiency.

2.80

DOE b(3)

APPENDIX

A2.01 The average pressure P in a spherical imploding core of radius R is defined by

$$\bar{P}V = \bar{P}(\tfrac{4}{3}\pi R^3) = \int_0^R P\,(4\pi r^2)\,dr, \qquad (2.29)$$

where P is the pressure at the distance r from the center; hence,

$$\bar{P} = \frac{3}{R^3} \int_0^R P r^2\,dr.$$

It follows, therefore, from equation (2.11), for a parabolic pressure distribution, that

$$\bar{P} = \frac{3P_0}{R^3} \int_0^R \left(1 - \frac{r^2}{R^2}\right) r^2\,dr. \qquad (2.30)$$

The integral in equation (2.30) may be evaluated as follows:

$$\int_0^R \left(1 - \frac{r^2}{R^2}\right) r^2\,dr = \int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr$$
$$= \frac{R^3}{3} - \frac{R^5}{5R^2} = \frac{2}{15}R^3.$$

If this result is inserted into equation (2.30), it is seen that

$$\bar{P} = \frac{2}{5}P_0. \qquad (2.31)$$

A2.02 Because of the pressure gradient in the core, the material will be accelerated, and this acceleration, $a(r)$, at the radial distance r, is given by the expression

$$a(r) = -\frac{1}{\rho}\frac{dP}{dr},$$

where ρ is the density of the core material. If P is again expressed by equation (2.11), then

$$a(r) = -\frac{P_0}{\rho}\frac{d\left(1 - \frac{r^2}{R^2}\right)}{dr} = \frac{P_0}{\rho}\frac{2r}{R^2}.$$

Hence, the acceleration, a, at the surface of the core, where $r = R$, is

$$a = \frac{2P_0}{\rho R},$$

so that

$$P_0 = \frac{\rho R a}{2}.$$

Upon inserting this expression into equation (2.31), it follows that

$$\bar{P} = \frac{1}{5}\rho R a,$$

which is equation (2.12).

Chapters 3-9
and the index
withheld in their
entirety